Stern, Climate Policy and Saving Rates*

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Abstract

Dasgupta and Weitzman have shown that the saving rates implied by the *Stern Review*'s values for the rate of pure time preference and the elasticity of the marginal utility of consumption are too high from either a normative (Dasgupta) or descriptive (Weitzman) perspective. Given the attention this debate has received in the literature, there is a need for a rigorous presentation of the determinants of saving rates in models actually used to evaluate climate change policy. This paper provides the first detailed investigation of the implications of Stern's parameter choices for saving, firstly in standard neoclassical growth theory and then in a widely used climate policy model based on that theory, Nordhaus's Dynamic Integrated model of Climate and the Economy (DICE). In theory and practice, optimal saving rates in the presence of near-zero pure time preference are far from the near-100 per cent ones obtained from simpler models used by several of the *Review*'s critics. We show that in DICE, for the utility function used in the *Stern Review,* optimal saving rates do not exceed 32 per cent, and that this falls to 26 per cent when using Stern's revised value for the elasticity of the marginal utility of consumption.

Introduction

Dasgupta (2007) and Weitzman (2007) have shown that the saving rates implied by the *Stern Review*'s (2007) values for the rate of pure time preference and the elasticity of the marginal utility of consumption are too high from either a normative (Dasgupta) or descriptive (Weitzman) perspective. While Weitzman (2007:723) agrees with Stern's conclusion if not his method, others such as Nordhaus (2008) maintain that the *Review*'s choice of utility function parameters drives the choice of fundamentally inappropriate climate policy.

Given the attention that these parameters and their associated saving rates have received in the climate change literature, there is a need for a rigorous presentation of the determinants of saving rates in models used to evaluate climate change policy. This paper provides the first detailed investigation[i] of the implications of Stern's parameter choices for saving, firstly in standard neoclassical growth theory and then in a widely used model for climate change policy evaluation based on that theory, Nordhaus's Dynamic Integrated model of Climate and the Economy (DICE). In theory and practice, optimal saving rates in the presence of near-zero pure time preference are far from the near-100 per cent ones obtained from simpler models used by several of the *Review*'s critics. We show that in DICE, for the utility function used in the *Stern Review,* optimal saving rates do not exceed 32 per cent, and that this falls to 26 per cent when using Stern's revised central value for the elasticity of the marginal utility of consumption.

The paper is structured as follows. Section 2 uses the theory of saving in the Ramsey model to show that saving rates using Stern's utility function parameters will be reasonable and explains and critiques claims of near-100 per cent saving rates under these parameters. Section 3 compares the saving rates obtained from DICE using Stern's and Nordhaus's parameters and also shows that the difference in saving rates makes a minimal contribution to the difference in optimal policy under the different social welfare functions. Section 4 concludes.

Before proceeding, it is important to address the meaning of 'reasonable' as applied to optimal saving rates. It was noted above that authors have criticised the level of optimal saving rates from both a positive and a normative perspective. The case against the relevance of the positive perspective has been well made by others (see Dietz *et al* 2008) so the focus here is on the normative interpretation of 'reasonable'. A normative perspective would permit any saving rate as reasonable as long as it did not place too great a burden on the poorest generation. This makes acceptable rates difficult to pin down exactly. While rates close to 100 per cent are obviously unreasonable for most income levels, rates in the order of 30 per cent of GDP are at least potentially reasonable.

Saving Rates in the Ramsey Model

This section outlines the theoretical determinants of saving rates in the kind of model used to evaluate climate change policy. In contrast to saving rates in a simpler model used by some of the *Review*'s critics, steady-state saving is bounded above by a capital share which is typically much less than one.

The macroeconomic model most often used for investigating the economics of climate change is the Ramsey model with iso-elastic utility and Cobb-Douglas production (for an exposition, see Barro and Sala-i-Martin 2004). In this model the steady-state saving rate $s*$ is given by (Barro and Sala-i-Martin 2004:107):

$$s* = \alpha \frac{(x+n+\delta)}{(\delta + \rho + \eta x)} \qquad (1)$$

where $0 < \alpha < 1$ is the share of capital in production, $x \geq 0$ is the rate of growth of total factor productivity (TFP), n is the growth rate of population, $\delta \geq 0$ is the rate of capital depreciation, $\eta > 0$ is the elasticity of the marginal utility of consumption and $\rho > 0$ is the rate of pure time preference.

During the transition to this steady state, the behaviour of the saving rate is governed by how agents react to changes in factor prices caused by changes in the volume of capital. Diminishing marginal returns ensure that if the economy starts with a capital stock lower than the optimum, the rate of return will fall as the economy approaches the optimal capital stock. This falling interest rate induces opposing income and substitution effects and in general the overall effect on transitional saving rates is ambiguous. However, in the Cobb-Douglas case the saving rate will rise, fall or stay constant for the entire transition and is given by (Barro and Sala-i-Martin 2004:108):

$$s = \frac{1}{\eta} - \psi \frac{\hat{k}^{\alpha-1}}{\alpha A} \qquad (2)$$

where $\psi \equiv \{(\delta + \rho + \eta x)/\eta\} - \alpha(x + n + \delta)\}$, so that they time path of saving

depends on whether ψ is positive, negative or zero.

Combining this theory with two additional facts about saving rates in the Ramsey model, we can gain a picture of the time path of saving under Stern's parameterisation of the social welfare function.

Fact 1 is that *steady-state saving rates are bounded above by* α (Barro and Sala-i-Martin: 2004:135). The standard limitation on household borrowing in the Ramsey model requires $(x + n + \delta) < (\delta + \rho + \eta x)$ in the steady state. As steady-state saving is given by $s^* = \alpha \dfrac{(x + n + \delta)}{(\delta + \rho + \eta x)}$, we must have $s^* < \alpha$, *regardless of the level of pure time preference or TFP growth*. For a standard capital share of 0.3, this means steady-state saving is less than 30 per cent, which is arguably reasonable. Fact 2 is that, *for* $s^* < \dfrac{1}{\eta}$ *the saving rate falls monotonically toward its steady-state value* (Barro and Sala-i-Martin 2004:136). Fact 1 implies this case holds for $\eta \in [1,3]$ and a standard capital share of 0.3. This range covers both $\eta = 1$, the value used in the *Stern Review*, and $\eta = 2$, the value Stern (2008:23) has said that he would use "with the value of hindsight".

Facts 1 and 2 tell us that climate policy models based on this standard Ramsey model and using Stern's utility function parameters would yield saving rates bounded above by 0.3 in steady state. Yet Arrow (1995:15), Dasgupta (2007:6) and Weitzman (2007:709) argue that low rates of pure

time preference generate incredibly high optimum saving rates.[ii] The key to reconciling these two facts is understanding that these authors assume a different and rather special production function in which the marginal product of capital is independent of how much capital the economy has accumulated. In this case, output is proportional to the capital stock, the production function is $Y = AK$ and marginal product of capital is constant at A. This absence of diminishing marginal returns is sometimes motivated as being consistent with a broad interpretation of capital which includes both physical and human assets and indeed many modern growth models include mechanisms that eliminate diminishing marginal returns to knowledge at the social level (see for example Romer 1986). However, Barro and Sala-i-Martin (2004:211-2) provide a simple example to show that this interpretation of K as 'composite' capital requires *all* of its components to have constant marginal products. This assumption is arguably unrealistic for physical capital (Ray 1998:82; Easterly 2001: Chapter 3) and it has a significant effect on the behaviour of the optimal saving rate, which is given by (Barro and Sala-i-Martin 2004:209):

$$s^* = \frac{1}{\eta A}(A - \rho + \eta n + (\eta - 1)\delta) \qquad (3)$$

Weitzman, Arrow and Dasgupta assume zero depreciation and population growth so this collapses to

$$s^* = \frac{r - \rho}{\eta r} \qquad (4)$$

We can now compare Equation (4) with the same expression for Cobb-Douglas production. Firstly, as the capital share of output in the AK model is 100 per cent, saving rates can become very high; Dasgupta's example of $r = 4$ per cent with the *Review*'s parameters of $\eta = 1$ and $\rho = 0.1$ percent generates a saving rate of 97.5 per cent. Dasgupta rightly describes this as absurdly high but does not focus on the role of constant returns to capital in creating this absurdity. Secondly, the fact that there is no transition in the AK model (Barro and Sala-i-Martin 2004:208) means that the optimum saving rate is constant, so an infinite number of generations will have to bear the high rates of saving that emerge with low rates of pure time preference. In contrast, for $s^* < \frac{1}{\eta}$ and Cobb-Douglas production, initial saving rates may be high, but do not continue indefinitely.

To summarise, altering the production function in the Ramsey model has important implications for the behaviour of the saving rate. The AK production function assumes constant returns to investment in all types of capital, but this seems particularly implausible for physical capital. Such a model is not therefore not well-suited for a central role in the debate about the accumulation of physical capital over time.

Saving rates in DICE

We now demonstrate the theoretical results above in a specific climate-economy model, Nordhaus's DICE. DICE is a seminal, publicly available Integrated Assessment Model (IAM) which links a macroeconomy populated with optimising agents to a climate model. While the IAM used in the *Review* employed a superior treatment of risk (Stern 2007:173-4), the underlying

macroeconomic model lacks microfoundations so cannot inform the debate about optimal saving rates and time preference.

While DICE differs from the standard Ramsey model, these differences (the existence of a production externality, the use of a finite time horizon, the fact that some exogenous variables are time-varying) do not have particularly sizeable effects on the resulting saving rates, so Equations (1) and (2) are good approximations to saving rates actually observed in DICE.[iii]

Figure 1 shows three paths for the saving rate produced by DICE under the 'base' case with no mitigation policy[iv], all of which display the behavior outlined in Section 2. The line s1 is from the DICE base run using the model's default parameters of $\eta = 2$ and an annual rate of pure time preference of 1.5 per cent. The line s2 shows the saving rate obtained when $\eta = 2$ (Stern's 'hindsight' choice) with the *Review*'s rate of time preference, (0.1 per cent per year). The third line shows the saving rate under the *Stern Review*'s original parameters of $\eta = 1$ and $\rho = 0.1$ per cent per year.

Figure 1: Saving rates in DICE07 base case
Different rates of pure time preference and η
(Figures at end)

Notes: s1: saving rate from DICE07 base run; s2: saving rate in base run with Stern 'hindsight' utility function parameters; s3: saving rate in base run with utility function parameters as in the *Stern Review*.

There are two points to note about these saving rates. The first is that rates using Stern's original parameters should not be rejected out of hand; they are at the very least far removed from the "patently absurd" rates Dasgupta (2007:6) calculates from the AK model. Secondly, as an increase in η

unambiguously lowers steady-state saving[v], raising η from one to two brings the optimal saving rate closer to the path that results from Nordhaus's choices for the social welfare function. Comparing a year in the mid-22nd century, the optimal saving rate in 2155 is 21 per cent under Nordhaus's parameters and 24 per cent (s2) or 26 per cent (s3) with Stern's. Of course the near-zero rate of time preference in both of Stern's parameterisations raises the saving rate relative to Nordhaus's paths. But comparing paths s1 and s2 over the entire model horizon (which is longer than the time series shown), the *maximum* difference between Stern's and Nordhaus's optimal rates is 4.5 percentage points. Given the large uncertainties surrounding IAM outputs (Stern 2007:164) this does not seem worth quibbling about.

A second way of demonstrating that the differences between these saving rates is small is by analysing their effects on the stock of capital, and hence on emissions and the optimal rate of mitigation. Figure 2 shows times series of the chosen rate of emissions control under four variations of the 'optimal' policy scenario in DICE:

1. Optimal policy under Nordhaus's default utility function parameters ($\rho = 1.5$ per cent and $\eta = 2$);

2. Optimal policy under Nordhaus's default utility function parameters and a fixed saving rate of 22 per cent;

3. Optimal policy under Stern's 'hindsight' utility function parameters and the fixed saving rate of 22 per cent; and

4. Optimal policy under Stern's 'hindsight' parameters ($\rho = 0.1$ per cent and $\eta = 2$).

Figure 2: Optimal emissions control rate in DICE07
Different utility function parameters and saving rates
(Figures at end)

Note: see text for description of scenarios.

Now, the level of the fixed saving rate in scenario (2) is chosen[vi] to be a close match to the endogenous saving rate in scenario (1), and fixing the saving rate permits the same capital stock to be accumulated, and hence the same amount of emissions to be produced, regardless of the social welfare function. We can therefore approximately decompose the shift in optimal mitigation policy caused by the move from Nordhaus's to Stern's utility function parameters into the two shifts from (2) to (3) and (3) to (4).

Because they have the same saving rates, capital stocks, and emissions, the difference between the chosen rates of emissions control in scenarios (2) and (3) is due to differences in how the agent values avoided damages from climate change, and represents the "welfare effect" of changing the utility function. Naturally the higher value of avoided future damages under Stern's utility function raises the optimal rate of control.

The difference in optimal emissions control between scenarios (3) and (4) approximates the "capital effect" of the shift to Stern's utility function: that is, that part of Stern's policy prescription caused by the fact that slightly higher saving rates result in slightly more capital and higher emissions. Figure 2 confirms that this capital effect is small, causing an average change of 1 percentage point in the optimal rate of emissions control.

Conclusions

The "singular disagreement" (Dietz *et al* 2008:10) about whether the pure rate of time preference and the elasticity of the marginal utility of consumption should be chosen according to positive or normative criteria is a normative question and, as such, no empirical result can settle it. However, this paper clarifies the terms under which the 'unreasonably high saving rates' argument can legitimately be used as a criticism of Stern's parameterisation of the utility function: those maintaining this criticism should make clear and defend their use of the AK production function. The paper also shows, in a specific IAM, that despite the very different approaches to parameterising the social welfare function, the optimal saving rates arising from Stern's parameters are really not much different from those arising from Nordhaus's choices. Once we move away from the AK model, the saving rates resulting from Stern's parameters must be bounded above by the capital share in steady state; furthermore, Stern's revised value for η ensures that even in the transition the optimal rate is around 25 per cent. While these rates would not satisfy Ken Arrow[vii] it should stop those with less stringent definitions of an unacceptable burden requiring that the *Review*'s saving rates (and by implication its utility function parameters and perhaps even its policy prescriptions) be rejected emphatically.
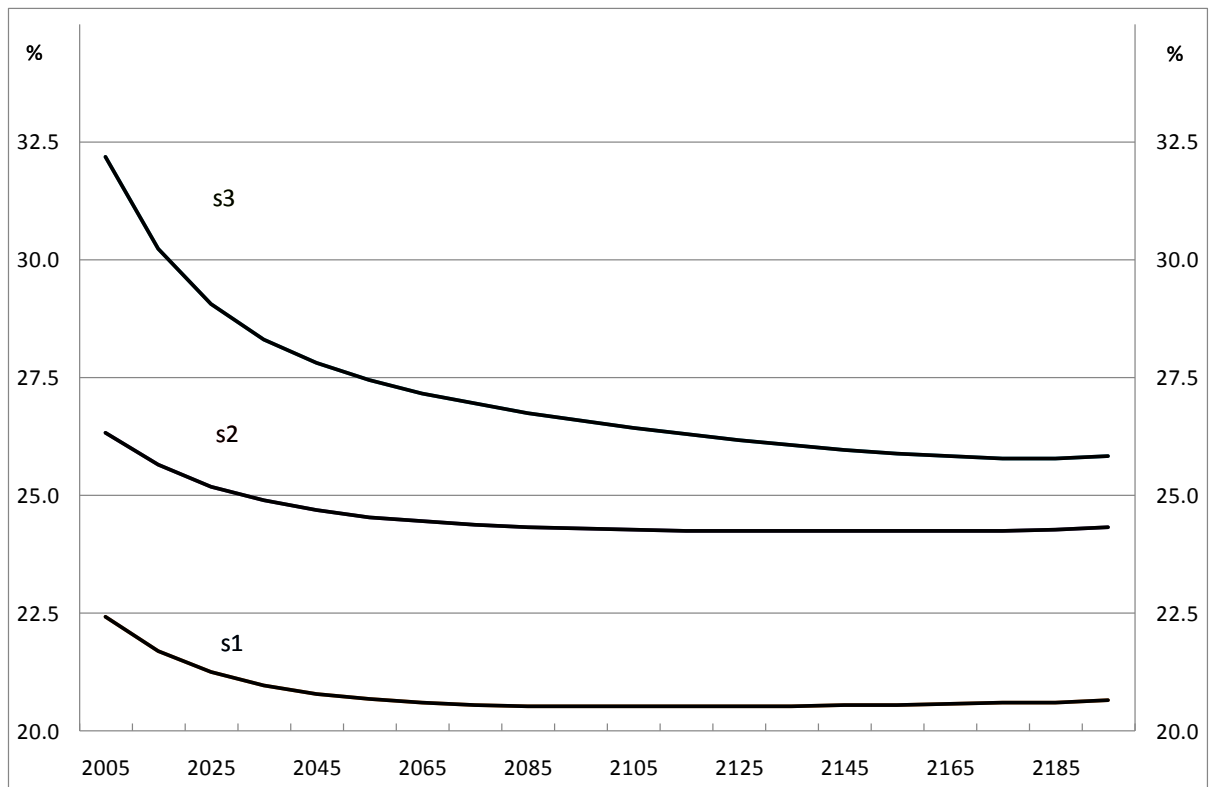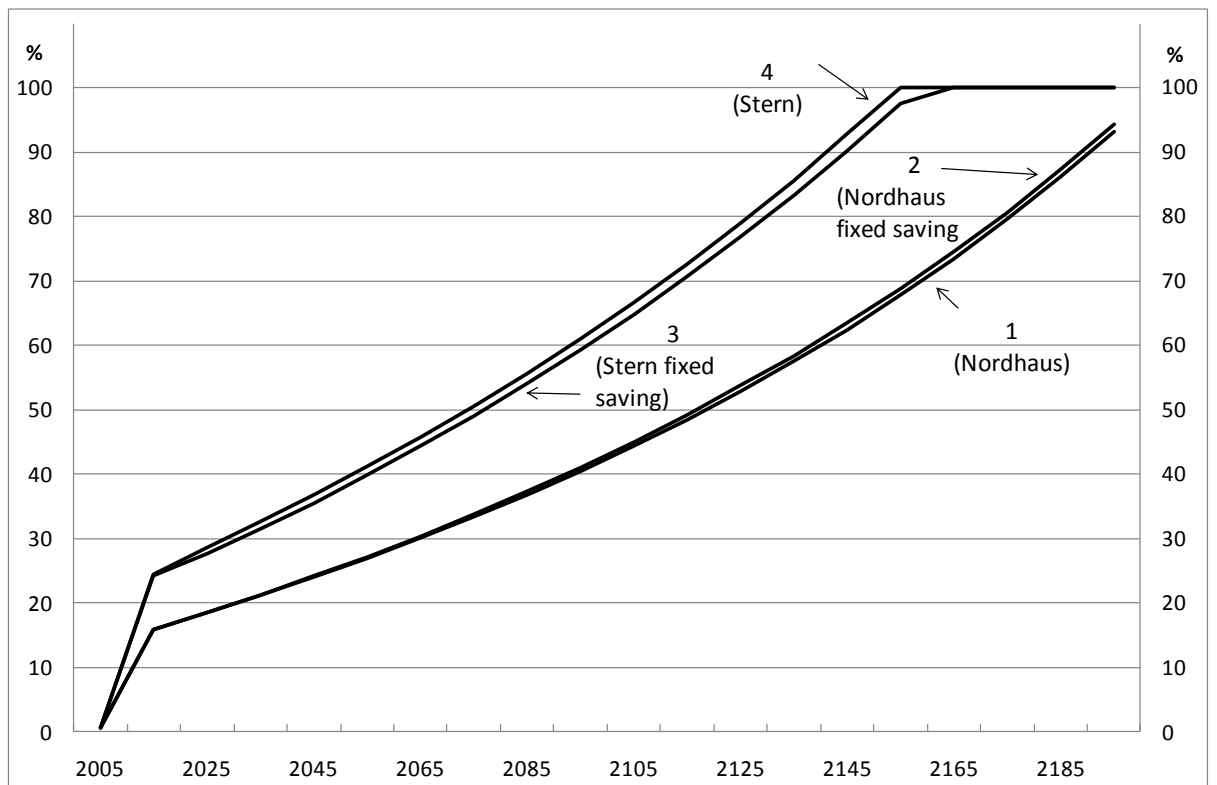
Figure 1

Figure 2

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Endnotes

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[i] There have been three brief responses to the saving rate criticism to date. Stern (2007:54) anticipated the criticism, noting in the *Review* that high optimal saving rates do not generalise to fully developed macroeconomic models featuring growth in total factor productivity and diminishing returns to factor inputs. Post-*Review* (2008), he provided a specific numerical example of this fact from Mirrlees and Stern (1972). DeLong (2006) also defended Stern's parameters, but did so by adding total factor productivity growth to a simple model without diminishing returns to physical capital.

[ii] Nordhaus (2008:179) also warns that Stern's utility function parameters will lead "at a first approximation" to a doubling of the optimal global net savings rate. It is unclear whether this figure comes from a specific model.

[iii] The production externality lowers the marginal product of capital. Given the levels of damages in IAMs, the size of this interest rate change is small in practice (Kelly and Kolstad 2001:144), so the effect of damages on saving rates is generally minimal and the expression for the steady-state saving rate without damages Equation (1) is a good approximation to the rate with damages. The fact that DICE is a finite- rather than an infinite-horizon Ramsey model implies that DICE will not attain the same steady state as an infinite-horizon model. However, for large T as in DICE, the initial part of this path will be close that of the infinite-horizon model (Barro and Sala-i-Martin 2004:105), so the steady-state saving rate is a good approximation for the rate in DICE before agents begin dissaving. The purpose of this paper – comparing different paths for saving rates, all of which use the same set of time-varying exogenous parameters as inputs – means that whether the rates of TFP and population growth are time varying or constant is irrelevant. Finally, the use of discrete, decadal time may also induce numerical differences between the results from the standard Ramsey model and DICE.

[iv] Optimal mitigation policy is actually introduced in the DICE07 base case after 250 years of inaction (Nordhaus 2008: 67-8).

[v] $\dfrac{ds*}{d\eta} = \dfrac{-x}{(\rho + \eta x + \delta)} s* < 0.$

[vi] By Nordhaus – the publicly available DICE07 programs contain the option of being run with this fixed saving rate.

[vii] His very strong argument (1995:14) against low rates of pure time preference requires the optimal saving rate to be morally acceptable for any "physically possible" production technology.