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Raghbendra ***Jha****, Hari K.* ***Nagarajan*** *& Kolumum R.* ***Nagarajan***

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Arndt-Corden Department of Economics
Crawford School of Public Policy
ANU College of Asia and the Pacific

Fiscal Federalism and Competitive Bidding for Foreign Investment as a Multistage Game

Raghbendra Jha
ANU
Canberra, Australia

Hari K. Nagarajan
NCAER,
New Delhi, India

Kolumum R. Nagarajan
CMI,
Chennai, India

Abstract

This paper models the behavior of states in a federal country wising to attract foreign firms to locate within their own individual jurisdictions. The essential intertemporal character of this decision is modeled as a multi-stage game to attract such foreign investment in these states. It is found that, when states with unequal political or economic infrastructure compete, the resulting Nash equilibrium profiles are inefficient. Under certain conditions, states that have won once, can "allow" a rival to win in a subsequent stage. The resulting Nash Equilibrium is more efficient. If the option of "allowing" a rival to win is not available, then states may resort to "suicide" strategies defined as outcomes created by history of losses.

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All correspondence to:
Prof. Raghbendra Jha,
Australia South Asia Research Centre,
College of Asia and the Pacific
H.C. Coombs Building (09)
Australian National University,
Canberra, ACT 0200, Australia
Phone: + 61 2 6125 2683; Fax: + 61 2 6125 0443; Email: r.jha@anu.edu.au

I. Introduction

Competition between sub-national governments in matters of tax and public expenditure has been an issue of major concern in public economics for long. Reflecting an early concern Gordon (1983) examined whether tax competition and public expenditure competition among sub-national units (henceforth states) that may not be overly concerned over the spillover effects of such competition across their respective geographic boundaries, have often been associated with the emergence of sub-optimal national outcomes.

While the above analysis concentrates on domestic issues clearly competition among states extends to offering concessions for attracting international investment – both portfolio and foreign direct investment (FDI). A concern of the early literature on this issue was whether there should be policy co-ordination across states when it comes to offering such concessions or, whether states should be given a free hand to pursue policies to attract firms. Thus, Kanbur and Keen (1993) and Wildasin (1989) argue that inter-state tax coordination rather than tax competition is welfare optimal. This is because tax competition to attract new firms could reduce to what Sinn (1994) calls "benefit taxation". However Brennan and Buchanan (1980) argue that tax competition is in fact "constitutionally efficient".

Another strand of the literature has explicitly incorporated the effects of business climate into models of inter-state competition for firms (Edwards and Keen, 1996, and Ellis and Rogers, 1999, and Venkatesan and Varma, 2000). Much of this analysis is conducted as a single stage prisoner's game.[1]

Whereas much of this literature has focused on competition for portfolio investment others have emphasized the need to distinguish between capital and firm mobility. Doyle and Wijnbergen (1984) and Bond and Samuelson (1986) have examined the location choice of a specific profit-making firm. King and Welling (1992), King, McAfee and Welling (1993), and, Besley and Seabright (1999) model competition between governments in a dynamic environment.

[1] Janeba (1998) has noted that if the cost functions of firms are independent of their or their rivals' locations then, given the tax differentials offered by the governments, the firm will locate in that state which offers the lowest tax or the highest subsidy. From the point of view of a typical state this is again a prisoner's dilemma outcome. In a similar vein Yang (1996) has shown that the equilibrium tax rate profile involves zero tax rates or, equivalently, the maximum subsidy possible.

Equally important is the literature that assesses the impact of competition among states for foreign direct investment (FDI).[2] States compete by offering tax concessions and other incentives to prospective investors. Such competition often resembles an arms race and is difficult to stop once started.[3] A state competes because of, i) a perceived increase in the welfare of its residents consequent upon the decision of firms to locate within its geographical area, and, ii) political expediency caused by the government's need to retain power beyond a specific time period.[4]

Recent literature has interpreted states' incentives for attracting firms more broadly to include all types of incentives for firms to locate within their boundaries and called it "competitive bidding" or simply "bidding" for FDI. Most of this literature underscores the deleterious effects of such bidding on sub-national and, hence, aggregate welfare. Thus, Nov (2006) argues that bidding for FDI by countries (not just states) represents a sub-optimal global solution and suggests that a global mechanism should be devised to ensure that FDI is allocated efficiently. In particular, he seeks an expanded role for the World Trade Organization (WTO) in regulating and overseeing FDI. This is the so-called WTO+ solution. Even more relevant for the case of developing countries Kessing et al. (2009) show that the existence of vertical fiscal inefficiencies prevents federal countries from successfully bidding for FDI. In particular, if these countries also have weak institutions their ability to attract FDI is lower in comparison to unitary countries.

In the present paper we examine the behavior of states within a federal framework where, competition between states takes place in order to attract investments from outside this system (i.e., country). Firms are offered inducements by competing states in the form of tax concessions and other incentives, all of which involve costs. A higher level of tax incentive offered to incoming firms in addition to possibly attracting these firms, provide a significant political advantage at home. This fact is then endogenous to the payoffs. Competition between states is then modelled as a stage game of three stages which corresponds to the planning period (in years) of elected governments. The paper discusses the case where there

[2] See for example, Hwang and Choe (1996), Yang (1996), Haufler (1998), Janeba (1998), and, Lorz (1998). Wilson (1999) is a recent survey of the literature.

[3] See Grady (1987), and, Jenn and Nourzad (1996).

[4] A related literature examines inter-jurisdictional (inter-state) competition in and analyses the welfare implications of such competition. Such welfare implications include extraction of surplus from incoming firms (Olsen and Osmundsen, 2000, and maximizing the revenue for the centralized authority (Keene and Kotsogiannis, 2003, and Cardarelli and Taugourdean, 2002).

is a series of investing firms that are homogeneous in every respect. The paper also examines the consequences of competition between states with different measures of political advantage derived from offering these incentives.

Our point of departure (and hence contribution) is twofold. The first is to capture the reputation effects of the states' offers over time and the consequence of competing for investing firms based partly on considerations of reputation building. Secondly, we explicitly recognize that the FDI decision process is an intertemporal one. The "stage game" envisaged in the extant literature is actually a two-stage game with state governments deciding to bid in the first stage and the firms reacting in the second. The game ends at this point. In our model, however, the firm's decision to locate is treated as exogenous. The states bid within a planning horizon. Hence there can be repeated winners – indicating the intertemporal nature of the process. This is a deviation from and a substantial generalization of the extant models.

In this paper we prove the following results:

a) All the Nash equilibria (N.E.) that result as an outcome of competition between states with unequal political or economic infrastructure will be inefficient.
b) Under certain conditions states that have won once, can "allow" a rival to win in a subsequent stage. The resulting Nash equilibrium will be a more efficient equilibrium. (This is a possibility only if all states involved resort to such a strategy.)
c) If the option of "allowing" a rival to win is not available, then states may resort to "suicide strategies" in order to overcome continuous losses.[5] Within a planning horizon (3 or 5 years) suicide bids are resorted to in either the second or the fourth stage of the game.

The paper is organized as follows. Section II describes the model as a stage game involving 2 states. Section III describes the theorems or results developed in this paper in a heuristic manner. We present certain general theorems for 3 stage games and describe all the possible Nash Equilibrium profiles. We also describe at length an outcome that we term as 'suicide strategies'. Suicide strategies constitute a way for a loser to win at any cost and bear resemblance to some aspects of economic liberalization. Section IV provides detailed proofs of the theorems. Section V examines the impact of unequal reputation functions on the Nash Equilibrium profiles and Section VI concludes.

[5] Suicide strategies are defined below.

II. The Model

To keep the model within tractable limits, we consider two states, P_1 and P_2 competing for a homogenous sequence of firms. The basic characteristics of the model are as follows.

1. States choose their bids (tax and other concessions) at any stage from a finite set $S = \{0, x_1, ..., x_n, x\}$, $0 < x_1 < ..... < x_n < x$. A bid pair ($\alpha, \beta$) means P_1 bids α and P_2 bids β, where α, β both belong to S.[6]

2. In the first stage the states choose their bids freely without any preconditions. The first stage bid pair is denoted by $\left(s_1^1, s_2^1\right)$ where $\left(s_1, s_2\right)$ are the contingency plans (as opposed to actual bids) of states P_1 and P_2 respectively.

3. The representative firm locates in P_1 if $s_1^1 > s_2^1$ and in P_2 if $s_2^1 > s_1^1$. In case the bids are equal, the firm locates in either of the states with equal probability of 0.5. We denote the outcome of a bids pair $\left(s_1^1, s_2^1\right)$ by the triple $\left(s_1^1, s_2^1, i\right)$ where $i = 1$ if P_1 wins and $i = 2$ if P_2 wins.

4. In the second stage of the game, the states are assumed to have full information of (i) the bids in the first stage and (ii) the winner in the first stage. The states then choose their second stage bids simultaneously. Thus, if we let H^1 denote the set of all histories h^1 at the end of the 1st stage, each state's contingency plan s_i^2, $i = 1,2$ is a function from the set H^1 into the set *S*.

   The move of the firm at the 2nd stage is based not only on the relative values of the offers of the states but also on the 1st stage history. Thus, at a first stage outcome h^1 where P_1 has won, if the second stage bids (α, β) are equal or, if $\alpha \geq \beta$ then the firm locates in P_1. If $\alpha < \beta$ then, it locates in P_2. Symmetrically, if P_2 has won in h^1, and the second stage's bids are α and β, the firm locates in P_2 if $\alpha \leq \beta$ and in P_1 if $\alpha > \beta$.

5. The game in subsequent stages proceeds in the same manner inductively. For example in the third stage, each state observes the history h^2 at the end of the second stage, (h^2 contains the information about the sequence of bids at the first two stages and the sequence of wins at the first two stages) and selects its third stage bid.

[6] Proofs of the theorems developed in this paper are based on the assumption that S contains only two elements, zero and x. The analysis of Nash Equilibrium under more general assumptions where S contains more than two elements, though more complicated, yields essentially similar qualitative results.

Let $w_i(h^2)$ denote the number of times P_i has won in h^2, and suppose P_1 bids α and P_2 bids β at the history h^2. The firm locates in P_1 if $\alpha > \beta$ and in P_2 if $\alpha < \beta$. If $\alpha = \beta$, the firm locates in P_1 if $w_1(h^2) > w_2(h^2)$ and in P_2 if $w_1(h^2) < w_2(h^2)$. If $\alpha = \beta$ and $w_1(h^2) = w_2(h^2)$ then P_1 and P_2 have an equal chance of winning the firm. The game proceeds inductively in subsequent stages.

6. We now define the payoffs for a strategy profile (s_1, s_2), for the 3-stage game G. Note that, the final histories of (s_1, s_2) may be more than one, each history h being a sequence of outcomes together with a probability $\pi(h)$ of its occurrence.

   We define $c(\bullet)$, as a monotonic increasing function in the space $[0, x]$ where $c(0) \geq 0$. Let $(\alpha, \beta, 1)$ be an outcome in the r^th^ stage. We define the r^th^ stage payoff for $(\alpha, \beta, 1)$ as

$$p_1^r(\alpha, \beta, 1) = \delta^{r-1}[(x-\alpha) + c(\alpha - \beta)],$$

$$\text{and,} \qquad p_2^r(\alpha, \beta, 1) = \delta^{r-1}[-c(\alpha - \beta)]$$

   where, δ is the discount factor.

   Similarly for an outcome $(\alpha', \beta', 2)$,

$$p_2^r(\alpha', \beta', 2) = \delta^{r-1}[(x-\beta') + c(\beta' - \alpha')],$$

$$\text{and,} \qquad p_1^r(\alpha', \beta', 2) = \delta^{r-1}[-c(\beta' - \alpha')].$$

A final history h is a sequence of these outcomes say (O_1, O_2, O_3) and has a probability of occurrence, say π. Then the payoff at h is the sum of $p_i(O_r)$ multiplied by the probability π. If (s_1, s_2) is a strategy profile we define $p_i(s_1, s_2)$ as the sum of p_i(h) where h varies over the set of all final histories h realized by the profile (s_1, s_2).

It is useful to explain the game into extensive form. A part of this game tree is shown in Figure 1.

Figure 1 here

WE now discuss some aspects of this model.

i) The function $c(\bullet)$:

The implementation of a tax or subsidy differential by governments has both a current and future impact, the sum of which is assumed to be positive. We use a positive valued monotonic increasing (reputation) function $c(\bullet)$ to capture this effect. It can be thought of as the positive impact on the policymakers (loosely thought to comprise of bureaucrats and politicians) in terms of their continuance as policymakers consequent upon their ability to create enhanced "business climate".

For a pair of bids $(\alpha, \beta)\,; \alpha > \beta$, $c(\alpha - \beta)$ measures the utility plus the reputation for the state by making a larger bid α. If we are looking at tax concessions $c(\alpha - \beta)$ will have the negative revenue effect plus the positive long-term benefits that the state considers desirable by offering the subsidy. We assume *c* to be a strictly increasing function and to take positive values in $[0, x]$.

ii) The tie-breaking rule:

When bids are equal it is reasonable to assign the win to any of the players with equal probability assuming there are no exogenous factors to favor any particular player. In the first stage of the game, the firm has already evaluated the bidding states as being equal in status and chooses the highest bidder. In case of a tie, the decision to locate in any particular state is assumed to be outside the purview of the game or equivalently each state has an equal chance of winning. In the subsequent stages, incoming firms look for any factors that will help them decide their choice, in case of a tie. We have modeled this by the rule "the winner at a history wins again in case of a tie". This appears to reflect the reality sufficiently closely; especially in states or cities or regions in an emerging economy which compete to attract firms from outside the country to locate and invest in their areas. Those states, which have a better record of having attracted foreign business, will have a starting advantage at any stage of the game.

iii) Discreteness of Choice:

The set S of strategies in our case is a finite set. In the literature on tax competition or competitive subsidies offered by firms (equivalent to a negative tax), the problem is usually modeled as a game where the set of choices or strategies is a continuum $[0, x]$ (where *x* is the maximum pre-agreed bid) or $[0, \infty)$. There are two points to note here. If we model economic variables as belonging to a closed or open subset of the reals, we effectively smoothen out the small jumps in the value of the functions and can use the methods of calculus of variations or maximum principle. Very often there are crucial and important differences between a discrete model vis-à-vis a continuous model for the same problem.[7] In the case of bidding for a firm it quickly transpires that there is a notable difference in the possible equilibrium strategies. For, in the case of a continuum $[0, x]$ of strategies, it is clear that each competing state can infinitesimally increase its subsidy offer and defeat its competitor, thereby pushing the bid as high as possible. The effect of this is that both will offer the maximum possible – equal to the states' valuation of the total worth of gaining the firm- exactly like the classical bidding game.

In this paper, the set S of strategies is finite. The game is modelled in such a way that the set of choices or strategies is a continuum $[0, x]$, where, x is the maximum bid (or, $[0, \infty]$). Consequently the problem is reduced to one of finding bids under constraints of welfare maximization, or revenue and tax maximisation. In this paper, the constraints are the utility (political gains) of attracting investments against the cost or subsidies that can be associated with the process of attracting these investments. The extant literature examines the problem as one of optimization; the current paper consequently addresses the same problem as a finite stage game with a finite set of strategies.

III. Statement of theorems and results

We assume that the set of strategies, has only two elements, 0 and *x*. We start with conditions under which a given profile can be a N.E. profile in a three stage game, and describe all the possible N.E final histories.

Theorem 1

In order that (s_1,s_2) should be a N.E. profile, the following are necessary.

(i) The first stage bids are equal, i.e. $s_1^1 = s_2^1$.

(ii) Let h be any history after the first stage and let $w_i(h)$ denote the number of times P_i has won in *h*. Then $s_1(h)=x$ if $w_1(h) < w_2(h)$ and $s_2(h)=x$ if $w_2(h) < w_1(h)$.

(iii) The third stage bids are equal i.e. $s_1^3 = s_2^3$.

(iv) The final histories that are possible under a NE profile are the following.

a) (x,x,1); (x,x,1); (x,x,1), and,
(x,x,2); (x,x,2); (x,x,2).

b) (0,0,1); (x,x,1); (x,x,1), and,
(0,0,2); (x,x,2); (x,x,2)

[7] One such example is the logistic curve as against the logistic equation.

c) $\left\{\begin{matrix}(0,0,1);(0,x,2);(0,0,1)\\(0,0,1);(0,x,2);(0,0,2)\end{matrix}\right\}$ and

$\left\{\begin{matrix}(0,0,2);(x,0,1);(0,0,1)\\(0,0,2);(x,0,1);(0,0,2)\end{matrix}\right\}$

d) Same as (c) except that the first stage bids are (x,x).

The possible Nash Equilibrium profiles given the conditions shown below in the extensive form.

Figure 2 here

The question arises whether there are N.E profiles that can in fact yield these final histories. The following theorems give the full answers.

Theorem 2

In the three stage game,

i) The strategy profile (τ_1, τ_2) where $\tau_1^r = \tau_2^r = x$ for $r = 1,2,3$ is a N.E. profile.

ii) The profile (τ_1, τ_2) where $\tau_1^1 = \tau_2^1 = 0$ and $\tau_1^r = \tau_2^r = x$ for $r = 2,3$ is a N.E. profile if and only if $c(x) + \delta c(0) + \delta^2 c(0) \leq x/2$.

It is pertinent to ask what will happen if the winner at stage one decides to bid 0 at stage two and lets its competitor win. Under certain conditions states that have won once can “allow” a rival to win in a subsequent stage. The resulting Nash equilibrium is a more efficient equilibrium (This is a possibility only if both states resort to such a strategy). The results of such bidding behavior are stated in Theorems 3 and 3a.

Theorem 3

Let both P_1 and P_2 bid 0 in the first stage. At any subsequent history *h*, either player bids 0 if it has more wins in *h* than its competitor, and bids *x* if it has fewer wins than its competitor. If both P_1 and P_2 have equal number of wins in *h* then they bid 0. The resulting strategy profile is a N.E. profile if and only if $c(0) + c(x) + \delta c(0) \leq (\delta x/2)$. We denote the profile by (σ_1, σ_2).

Theorem 3a

Let both players bid *x* in the first stage, and follow the strategy described in Theorem 3 for the second and third stages. The resulting profile is a N.E. profile if and only if $c(0) + c(x) + \delta c(0) \leq \frac{\delta x}{2}$. We denote this profile by $(\overline{\sigma_1}, \overline{\sigma_2})$.

Remark 3.1

Note that, given the option of (σ_1,σ_2) in theorem 3 and the profile $(\overline{\sigma_1},\overline{\sigma_2})$ defined in 3a, players will prefer (σ_1,σ_2) since there is a likelihood of higher payoff arising from the first stage move.

The condition under which (σ_1,σ_2) is a N.E. profile, is fairly strong.

Even when δ is very nearly equal to 1. The condition is $c(0)+c(x)+c(0)\le(x/2)$.

Further since $c(x)+\delta c(0)+\delta^2 c(0)\le c(x)+c(0)+\delta c(0)$, the conditions under which (σ_1,σ_2) is a N.E. profile is sufficient to ensure that the profiles (s_1,s_2) and (τ_1,τ_2) in theorem2 are also N.E. profiles.

Remark 3.2

Given the choice of playing (s_1,s_2) as in theorem 2 and (σ_1,σ_2) as in theorem 3, what profile would the states choose, is now a question to be addressed. Playing (s_1,s_2), the winner at stage 1 will continue to win in subsequent stages. It is assured of a payoff $=x+c(0)+\delta c(0)+\delta^2 c(0)$ (if $s_1^1=s_2^1=0$), or $c(0)+\delta c(0)+\delta^2 c(0)$ if $s_1^1=s_2^1=x$. On the other hand playing (σ_1,σ_2), the winner at the first stage has the likelihood (with probability ½) of gaining a payoff equal to

$$x+c(0)-\delta c(x)+\delta^2(x+c(0))$$

$$=(1+\delta^2)x-\delta(c(x)-\delta c(0))+c(0)$$

which is larger than $x+c(0)+\delta c(0)+\delta^2 c(0)$ since $\delta^2 x>\delta c(0)+\delta c(x)$

$$(c(0)+c(x)+\delta c(0)\le\delta x/2)$$

and, a payoff $x+c(0)-\delta c(x)-\delta^2 c(0)$ with a probability of ½.

The loser at the 1st stage (in s_1,s_2) faces a maximum loss of $-c(0)-\delta c(x)-\delta^2 c(0)$ and if (σ_1,σ_2) is played, the loser faces a maximum loss $=-c(0)+\delta c(x)-\delta^2 c(0)$.

To put it differently, in (s_1,s_2), the winner at the 1st stage is assured of $x+c(0)+\delta c(0)+\delta^2 c(0)$ and in (σ_1,σ_2) it is assured of (at least) $x+c(0)-\delta c(x)-\delta^2 c(0)$. So if the states want to play safe they will choose (s_1,s_2).

On the other hand, the losers face $-c(0)-\delta c(x)-\delta^2 c(0)$ and $-c(0)+\delta c(x)-\delta^2 c(0)$ by playing (s_1,s_2) and (σ_1,σ_2) respectively. They will be sure to cut their losses by $2\delta c(x)$ by playing (σ_1, σ_2). So if the states decide to play in such a way that possible losses be minimized then they will choose to play (σ_1, σ_2).

Theorems 3 and 3a indicate the possibility that states may bid 0 in the third stage. This is a remarkable result. In fact, suppose the first stage bid pair is $(0,0)$ with two outcomes $(0,0,1)$ and $(0,0,2)$ such that either state may win with equal probability. In the second stage P_2 will bid *x* and P_1 will bid 0 at the outcome $(0,0,1)$ and the reverse at the outcome $(0,0,2)$. Thus at the end of the second stage P_1 and P_2 will have a win each in all cases and consequently will bid 0 at the third stage.

The question that naturally arises is whether this profile will give a better payoff to either player than the ones discussed in Theorem 2. The answer is yes. For instance, the payoff for the strategy in Theorem 3 is $x/2(1+\delta^2)$ which is greater than the payoff the strategy in Theorem 2(ii), which is x/2. The condition, under which the profile is an equilibrium profile, is much stronger than for those in theorem 2. The smaller the δ, the lower are the chances that this will be true, if states value their reputation at not too low a level. Another reason why this may be expected is because the first stage winner can always continue to win by matching the bid of the competitor. The incentive to deliberately lose in the second stage arises from the expectation of winning at the 3rd stage and earning a higher payoff. This cannot happen if the discount rate is too small. Winning at a later (third) stage will yield only a small current value with a probability ½, and will not offset the loss in reputation in the second stage given that $c(0)$ and $c(x)$ are not correspondingly very small.

The strategy *(s_1, s_2)* in theorem 2 yields the striking conclusion that if P_1 wins in the first stage then P_1 will continue to win in all the subsequent stages. At the end of the third stage P_1 will have a positive payoff whereas P_2 will have a net negative pay off. The strategies (σ_1, σ_2) of theorem 3 which yield the outcomes $(0,0,1;0,x,2;0,0)$ and, $(0,0,2;x,0,1;0,0)$ are of a different kind. Though the loser bids *x* in both strategies, now the winner decides to let the competitor win and take a chance of winning with a larger payoff in the third stage. We note of course, that the strategy (σ_1, σ_2) is an equilibrium profile only if $c(0)+c(x)+\delta c(x)\leq \delta x/2$. On the other hand *($s_1$, s_2)* is an equilibrium profile under the condition $c(x)++\delta c(0)+\delta^2 c(0)\leq x/2$. The condition for (σ_1, σ_2) is stronger, in the sense that the states' perception of the values of $c(0)$ and $c(x)$ are much lower in this case. This makes sense: since essentially, in the strategy (σ_1, σ_2), the loser plays the same strategy as in *(s₁, s₂)*, while the winner allows the competitor to win in the 2nd stage, because it can afford to gain a negative reputation in the second stage.

Suppose the condition $c(0)+c(x)+\delta c(0) \le \delta x/2$ is not satisfied while $c(x)+\delta c(0)+\delta^2 c(0) \le x/2$. Then (σ_1, σ_2) is not an equilibrium while (s_1, s_2) is. In this situation states face the possibility of a continual loss.

A way to remedy this situation will be for the loser in the first stage to take the lead and make a bid at the second stage that cannot be matched by its competitor, i.e., it should bid beyond x and hope its competitor will not follow suit.

Let us suppose that the set of strategies for each player is now $\{0, x, 2x\}$.[8] The payoff function is the same as before. For example if P_1 bids x and P_2 bids $2x$, then P_1 loses and receives a payoff of $c(2x-x) = -c(x)$ while P_2 wins and receives a payoff of $(x-2x)+c(2x-x) = -x+c(x)$.

At first sight this looks like a very unattractive choice for P_2. However, it has the effect of balancing out the winning history at any stage. In fact we prove the following. Let T be the 3-stage game with the following rules. There are two players P_1 and P_2 each with the strategy set $\{0, x, 2x\}$. The payoffs and outcomes are just as in the earlier 3-stage game. The function c is now defined for $2x$ also and we assume $0 < c(0) < c(x) < c(2x)$ and $c(x) \le x/2; c(2x) \le x$. We then have Theorem 4. We term the strategies in this theorem as suicide strategies.

Theorem 4 (Suicide Strategies)

Let $s = (s_1, s_2)$ be a strategy profile where,

(i) $s_1^1 = s_2^1 = 0$

(ii) s_1^2 takes the value x if P_1 wins in a first stage outcome and takes the value $2x$ if P_1 loses at a first stage outcome.

(iii) s_2^2 takes the value x if P_2 wins at a first stage outcome and takes the value $2x$ if P_2 loses at a first stage outcome.

[8] The value *2x* is not important. We have chosen *2x* since we assume that bids are to be multiples of *x*.

(iv) s_1^3 takes the value x if P_1 has won twice and takes the value $2x$ if P_2 has won twice, and is 0 if P_1 and P_2 each have won once.

(v) s_2^3 takes the value x if P_2 has won twice and takes the value $2x$ if P_1 has won twice, and is 0 if P_1 and P_2 each have won once.

Then s is a N.E. profile of the game T if and only if $c(0)+\delta c(0)+c(x)\geq x(1-\delta/2)$

Both players bid 0 at the first stage. At the second stage, the first stage winner bids x and the first stage loser bids $2x$. If h is any second stage history where both P_1 and P_2 have a win each, then both bid 0 at the 3rd stage. For all other 2nd stage histories, the winner bids x and the loser bids $2x$ at the third stage. The resulting profile will be Nash equilibrium if and only if the inequality $c(0)+\delta c(0)+c(x)\geq x(1-\delta/2)$ holds.[9]

The extensive form representation of the N.E. final history and the sucide strategy is shown in Figure (3).

Figure 3 here

Remark 4.1

The first stage outcomes for the strategy profile s defined in the theorem are $(0,0,1)$ and $(0,0,2)$. The second stage outcomes are $(0,0,1;x,2x,2)$ and $(0,0,2;2x,x,1)$. In the third stage $s_1^3(0,0,1;x,2x,2)=0=s_2^3$ and $s_1^3(0,0,2;2x,x,1)=0=s_2^3$ since both the second stage outcomes have one win for P_1 and one win for P_2. Thus in the third stage both P_1 and P_2 will again bid 0 and if the game continues for a further stage, both players have an equal chance of winning just like in the 1st stage. Thus in each odd number of stages, there is a level playing field where both players would bid 0.

[9] One can understand this inequality as follows. States decide to play the suicide strategy only if in their calculation any loser at the first stage can gain by it. For instance, suppose both play 0 at the first stage. Then either player has an equal likelihood of losing and faces further losses in the 2nd and the 3rd stages. In such a contingency the loser faces a cumulative loss of $-c(0)-\delta c(0)-\delta^2 c(0)$. On the other hand if the loser at the first stage (which can be either P_1 or P_2) opts for the suicide strategy, then it will play $2x$ at the 2nd stage and 0 at the third stage. In this case, either player faces a (possibly negative) payoff $-c(0)+\delta c(x)-\delta x$ at the end of the 2nd stage, and hopes to offset this by a positive payoff $\delta^2\ x/2$ in the 3rd stage. This totals to $-c(0)+\delta c(x)-(\delta x)(1-(\delta/2))$. Thus, states will opt for suicide strategies only if $-c(0)-\delta c(x)-\delta x(1-\delta/2)\geq -c(0)-\delta c(0)-\delta^2 c(0)$ or equivalently $c(0)+c(x)+\delta c(0)\geq x(1-(\delta/2))$.

IV. Proofs of Theorems

Proof of Theorem 1

We first take up (iii).

Proof of (iii)

Suppose $w_1(h) \neq w_2(h)$. We may assume that $w_1(h) = 2$ and $w_2(h) = 0$. At the third stage, if P_1 bids 0, P_2 wins by bidding *x* and loses by bidding 0. So P_2 bids *x*. Again, if P_1 bids *x*, then P_2 loses anyway but has better payoff by bidding *x*. Thus if $w_1(h) = 2$, $s_2^3(h) = x$. Once this is settled, because (s_1, s_2) is a N.E., P_1 must also bid *x*. Therefore $s_1^3(h)$ is also equal to *x*.

Let us consider the case $w_1(h) = w_2(h) = 1$. Suppose $s_1^3(h) = x$, $s_2^3(h) = 0$. Then the payoff for P_2 at the final outcome is $p_2(h) - \delta^2 c(x)$ while, $s_1^3(h) = x$, $s_2^3(h) = x$ yields two final outcomes $(h; x, x, 1)$ and $(h; x, x, 2)$ with payoff equal to $p_2(h)$. Hence if $s_1^3(h) = x$ then $s_2^3(h) = x$. By symmetry, if $s_2^3(h) = x$ so is $s_1^3(h)$ equal to *x*.

Proof of (ii)

Let $(s_1^1, s_2^1, 1) = h$ be an outcome at the first stage for the N.E. profile (s_1, s_2). There are two possibilities for s_1^2, namely $s_1^2(h) = x$ or $s_1^2(h) = 0$. Consider first the case $s_1^2(h) = 0$. We shall show that if $s_2^2(h) = 0$ then (s_1, s_2) is not a N.E. Since P_1 has won in *h* and in the second stage P_1 and P_2 both bid 0, which means that P1 wins again. From (ii), $s_2^3 = x = s_1^3$. That is, the outcome is $(h;0,0,1;x,x,1)$ with payoff for $P_2 = p_2(h) - \delta c(0) - \delta^2 c(0)$. If instead, $s_2^2(h) = x$, then P_2 wins at the second stage. By part ii) the third stage play must be $s_1^3 = 0 = s_2^3$ or $s_1^3 = x = s_2^3$ at the second stage outcome $(h;0,x,2)$. The final outcomes are either $(h;0,x,2;0,0)$ or $(h;0,x,2;x,x)$. In either case the payoff of P_2 is strictly greater than $p_2(h) - \delta c(0) - \delta^2 c(0)$. Therefore, $(s_1, s_2)$is not Nash equilibrium. A similar analysis shows that, in case $s_1^2(h) = x$ then also $s_2^2(h) = x$. This proves (ii).

Proof of (i)

Suppose on the contrary $s_1^1 = x$ and, $s_2^1 = 0$ This yields a unique first stage outcome (x, 0,1). By what was proved above, $s_2^2(x,0,1) = x$ while $s_1^2(x,0,1)$ may be 0 or x. In case $s_1^2(x,0,1) = 0$, We allow P_1 to deviate from s_1 by playing t_1, where t_1^1=0 and $t_1^2 = t_1^3 = x$. It turns out that $p_1(t_1, t_2)$ is strictly greater than $p_1(t_1, t_2)$. If on the other hand $s_1^2(x,0,1) = x$, we allow P_2 to deviate from s_2 by playing t_2 where $t_1^1 = t_2^2 = t_2^3 = x$. It is easily checked that $p_1(s_1, t_2) > p_2(s_1, s_2)$.

Proof of (iv)

As a consequence of (i), (ii) and (iii), it is easy to see that the possible N.E. final histories are, (a), (b), (c), (d) and also (c') (which is the same as (c) except that the third stage bid pair is (x, x)) and d' (which is the same as *d* except that the third stage bid pair is (x, x)). We can show that neither (c') nor (d') can be N.E. final history. For instance, consider (c'). The final history is,

$$\begin{Bmatrix} (0,0,1);(0,x,2);(x,x,1) \\ (0,0,1);(0,x,2);(x,x,2) \end{Bmatrix} \text{ and}$$

$$\begin{Bmatrix} (0,0,2);(x,0,1);(x,x,1) \\ (0,0,2);(x,0,1);(x,x,2) \end{Bmatrix}$$

Consider the payoff of P_1. It is $p_1 = x/2$. We can find an alternative strategy for P_1 which will yield a higher payoff. The deviation is that P_1 continues with the first stag bid as before. But bids x at all subsequent histories. Then, the final history (since P_2 continues with the original strategy) is,

$$\{(0,0,1);(x,x,1);(x,x,1)\} \text{ and}$$

$$\begin{Bmatrix} (0,0,2);(x,0,1);(x,x,1) \\ (0,0,2);(x,0,1);(x,x,2) \end{Bmatrix}$$

The payoff of P_1 is

$$1/2\left(x + c(0) + \delta c(0) + \delta^2 c(0)\right) + 1/2\left(-c(0) + \delta c(x) + \delta^2 c(0) - c(0) + \delta c(x) - \delta^2 c(0)\right)\Big)$$

$$= 1/2\left(x + c(0) + \delta c(0) + \delta^2 c(0)) - c(0) + \delta c(x)\right)$$

$$= 1/2\left(x + \delta c(0) + \delta c(x) - \delta^2 c(0)\right) > x/2$$

Hence (c') cannot be a N.E. final history. Similarly (d') cannot be a N.E. final history. These outcomes are shown in Figure 4.

Figure 4 here

Proof of Theorem 2

We prove (ii) in detail first. In order to show that (s_1, s_2) is a N.E. profile it is necessary and sufficient to show that $p_1(t_1, s_2) \le p_1(s_1, s_2)$ for all alternative strategies t_1 for P_1. Similarly for P_2. Because of symmetry, it is enough to consider deviations from s_1 and payoffs for P_1.

At the outset, note that $p_1(s_1, s_2) = x/2$. Suppose $t_1^1 \ne s_1^1$, so that $t_1^1 = x$. We consider all the possible final outcomes and payoffs and show that the corresponding payoffs are $\le p_1(s_1, s_2)$.

a) $t_1^1 = x$, first stage outcome is $(x,0,1)$, $t_1^2(x,0,1) = 0$; second stage outcome is $(x,0,1;0,x,2)$ and $t_1^3(x,0,1;0,x,2) = 0$. Final outcome is $(x,0,1;0,x,2;0,x,2)$. The payoff for $\mathrm{P}_1 = c(x) - \delta c(x) - \delta^2 c(x) \leq x/2$.

b) $t_1^2(x,0,1) = 0$, $t_1^3(x,0,1;0,x,2) = x$.

Hence the final outcome is $(x,0,1;0,x,2;x,x)$ and the associated payoff $= c(x) - \delta c(x) \leq x/2$.

c) $t_1^2(x,0,1) = x$; the second stage outcome is now $(x,0,1;x,x,1)$. Let $t_1^3(x,0,1;x,x,1) = 0$ so that the final outcome is $(x,0,1;x,x,1;0,x,2)$. The payoff is $c(x) + \delta c(0) - \delta^2 c(x) < x/2$, since by assumption, $c(x) + \delta c(0) + \delta^2 c(0) \leq x/2$.

d) $t_1^2(x,0,1) = x$, $t_1^3(x,0,1;x,x,1) = x$ gives the final outcome $(x,0,1;x,x,1;x,x,1)$ and P_1 payoff $= c(x) + \delta c(0) + \delta^2 c(0) \leq x/2$ by assumption. Thus we have proved that if $t_1^1 \neq s_1^1$, then $p_1(t_1, s_2) \leq p_1(s_1, s_2)$.

It is therefore enough to consider strategies t_1 where $t_1^1 = s_1^1 = 0$. The first stage outcomes are $(0,0,1)$ and $(0,0,2)$. We again consider all the possible final outcomes and payoffs when t_1 deviates from s_1 at the second or third stages.

a') Suppose $t_1^2(0,0,1) = 0$ and $t_1^2(0,0,2) = 0$. The second stage outcomes are respectively $(0,0,1;0,x,2)$ and $(0,0,2;0,x,2)$. For each of these, there can be two choices for t_1^3. However it is enough to consider the choice for which payoff p_1 is maximum. Since $s_2^3 = x$, the optimal bid for P_1 will also be x at all the second stage outcomes, that are realized by playing a given t_1^1 and t_1^2. Thus we need to consider only the pair of outcomes $(0,0,1;0,x,2;x,x)$ and $(0,0,2;0,x,2;x,x,2)$.

The payoff $p_1(t_1, s_2) = 1/2\,(x + c(0) - \delta c(x)) + 1/2\,(-c(0) - \delta c(x) - \delta^2 c(0))$

$$= 1/2\,(x - 2\delta c(x) - \delta^2 c(0)) \leq x/2 .$$

b') $t_1^2(0,0,1) = 0$ and $t_1^2(0,0,2) = x$. The second stage outcomes are $(0,0,1;0,x,2)$ and $(0,0,2;x,x,2)$. We need consider only the pair of third stage outcomes - $(0,0,1;0,x,2;x,x)$ and $(0,0,2;x,x,2;x,x,2)$. It is easily seen that the payoff is less than $x/2$.

c') $t_1^2(0,0,1) = x$ and $t_1^2(0,0,2) = 0$. The second stage outcomes are $(0,0,1;x,x,1)$ and $(0,0,2;0,x,2)$. We need consider only $(0,0,1;x,x,1;x,x,1)$ and $(0,0,2;0,x,2;x,x,2)$.

The payoff $=1/2\left(x+c(0)+\delta c(0)+\delta^2 c(0)\right)+1/2\left(-c(0)-\delta c(x)-\delta^2 c(0)\right)$

$$= x/2+\delta/2\left(c(0)-c(x)\right)< x/2$$

This completes the proof of (ii), since necessity of the condition follows from the working in (c).

The proof of (i) is along the same lines. At the outset, note that $p_1(s_1,s_2)=0$. We again consider deviation t_1 of s_1 and take up the deviation at the first stage. $t_1^1\neq s_1^1$. Hence, $t_1^1=0$. The first stage outcome is $(0,x,2)$. This implies that P_1 will continue to lose at stage two and three, whatever its move is, since P_2 bids x. Thus, whatever be t_1^2 and t_1^3, $p_1(t_1,s_2)<0$.

Hence deviation at the first stage cannot increase the payoff. We can then assume $t_1^1=s_1^1=x$ and consider deviations at the second stage. The proof is essentially the same as for (ii).

Proof of Theorem 3

Let (σ_1,σ_2) be the profile defined in the theorem. The first stage bids are $(0,0)$ with two outcomes $(0,0,1)$ and $(0,0,2)$. By assumption, $\sigma_1^2(0,0,1)=0$, $\sigma_2^2(0,0,1)=x$, $\sigma_1^2(0,0,2)=x$ and $\sigma_2^2(0,0,2)=0$. Hence the final outcomes are $(0,0,1;0,x,2;0,0)$ and $(0,0,2;x,0,1;0,0)$. The calculated payoffs are:

$$p_1(\sigma_1,\sigma_2)=1/2\left(x+c(0)-\delta c(x)+\delta^2(x/2)\right)+1/2\left(-c(0)+\delta c(x)+\delta^2(x/2)\right)$$
$$= x/2\left(1+\delta^2\right)$$

The proof that σ_1,σ_2 is Nash equilibrium is on the same lines as in Theorem 2 and is now sketched here.

First we can prove that neither P_1 nor P_2 can strictly improve its payoff by deviating in stage 1. We assume that t_1 is an alternative strategy for P_1. Let $t_1^1\neq\sigma_1^1$. This means that $t_1^1=x$, and the first stage outcome is $(x,0,1)$ since P_2 continues to bid 0. Now there are two cases $t_1^2(x,0,1)=0$ or $t_1^2(x,0,1)=x$. The corresponding 2[nd] stage outcomes are $(x,0,1;0,x,2)$ and $(x,0,1;x,x,1)$. After $(x,0,1;0,x,2)$ P_2 bids 0 and the better bid of P_1 is 0. The payoff at the final outcome $(x,0,1;0,x,2;0,0)$ is $=c(0)-\delta c(x)+\delta^2(x/2)< x/2\left(1+\delta^2\right)$, since $c(0)\leq x/2$ by assumption.

Other cases can be similarly examined by considering only those strategies t_1 where $t_1^1=0$.

The first stage outcomes are $(0,0,1)$ and $(0,0,2)$. Let $t_1^2(0,0,1)=x$ and $t_1^2(0,0,2)=x$. The second stage outcomes are $(0,0,1;x,x,1)$ and $(0,0,2;x,0,1)$. The best strategies for P$_1$ at the third stage are, $t_1^3(0,0,1;x,x,1)=x$ and $t_1^3(0,0,2;x,0,1)=0$, since P$_2$ continues to play σ_2.

The payoff at the final outcome is

$$=\frac{1}{2}\left(x+c(0)+\delta c(0)+\delta^2 c(0)\right)+\frac{1}{2}\left(-c(0)+\delta c(x)+\delta^2\frac{x}{2}\right)$$

$$=\frac{x}{2}+\frac{\delta^2 x}{4}+\frac{\delta}{2}\left(c(0)+c(x)+\delta c(0)\right)$$

$$=\frac{x}{2}\left(1+\delta^2\right)-\frac{\delta^2 x}{4}+\frac{\delta}{2}\left(c(0)+c(x)+\delta c(0)\right)$$

$$\leq\frac{x}{2}\left(1+\delta^2\right) \text{ since by assumption } c(0)+c(x)+\delta c(0)<\frac{\delta x}{2}$$

The other cases can be similarly dealt with. We omit the details. This proves the theorem. Incidentally, this also proves the necessity of the condition.

Proof of Theorem 3a

The final outcome is $\left\{\begin{matrix}(x,x,1;0,x,2;0,0)\\ and\\ (x,x,2;x,0,1;0,0)\end{matrix}\right\}$ and payoff $=\dfrac{\delta^2 x}{2}$.

Consider first, the deviation at the first stage

$t_1^1=0:\ t_1^2(0,x,2)=0$

The outcome is $(0,x,2;0,0,2;0,0,2)$ and $p_1<0$. If, on the other hand,

$t_1^1=0:\ t_1^2(0,x,2)=x$

The outcome is $(0,x,2;x,0,1;0)$: and, $p_1=-c(x)+\delta c(x)+\dfrac{\delta^2 x}{2}<\dfrac{\delta^2 x}{2}$

If we assume $t_1^1=x:\quad t_1^2(x,x,1)=x,\quad t_1^2(x,x,2)=x$ and,

the outcome is $(x,x,1;x,x,1;x,x,1)$ and $(x,x,2;x,0,1;0,0)$

$$p_1=\frac{1}{2}\left[c(0)+\delta c(0)+\delta^2 c(0)-c(0)+\delta c(x)+\delta^2\frac{x}{2}\right]$$

$$=\frac{1}{2}\left[\delta c(0)+\delta c(x)+\delta^2 c(0)+\delta^2\frac{x}{2}\right]$$

$$\leq \frac{1}{2}\left[\delta.\frac{\delta x}{2}+\frac{\delta^2 x}{2}\right]$$

$$\leq \frac{\delta^2 x}{2}$$

If we assume $t_1^1 = x$; $t_1^2(x,x,1) = x$, $t_1^2(x,x,2) = 0$ then the outcome for this stage is $(x,x,1;x,x,1;x,x,1)$ and $(x,x,2;0,0,2;0,0,2)$

$$p = \frac{1}{2}\left[c(0)+\delta c(0)+\delta^2 c(0)-c(0)-\delta c(0)-\delta^2 c(0)\right]$$

$$= 0 < \frac{\delta^2 x}{2}$$

Assume, $t_1^1 = x$; $t_1^2(x,x,1) = 0$, $t_1^2(x,x,2) = 0$, then the outcome is $(x,x,1;0,x,2;0,0)$ and $(x,x,2;0,0,2;0,0,2)$

$$p = \frac{1}{2}\left[c(0)-\delta c(x)+\frac{\delta^2 x}{2}-c(0)-\delta c(0)-\delta^2 c(0)\right]$$

$$= \frac{1}{2}\left[\frac{\delta^2 x}{2}-\delta c(x)-\delta c(0)-\delta^2 c(0)\right]$$

$$< \frac{\delta^2 x}{2}$$

This completes the proof.[10]

Proof of Theorem 4

It is easily verified that $p_1(s_1,s_2)$ = $p_2(s_1,s_2)$ $= x/2\left(1-\delta+\delta^2\right)$. In order to prove that (s_1,s_2) is a N.E. profile; we first take up deviations from s_1 in the first stage. Suppose P_1 bids x instead of 0 at the first stage. We assume that P_2 sticks to s_2, and see if P_1 can improve its payoff by its new move. The first stage outcome will now be $(x,0,1)$. By assumption, $s_2^2(x,0,1) = 2x$. We list out all possible outcomes and the corresponding payoffs for P_1 for a strategy profile (t_1,s_2), where $t_1^1 = x$ and t_1^2, t_1^3 arbitrarily vary. We then find out if any of these payoffs are strictly greater than $p_1(s_1,s_2)$.

There are three choices for $t_1^2(x,0,1)$; $0,x$ or $2x$. Correspondingly there are three outcomes at

[10] Note that, given the option of (σ_1,σ_2) in theorem 3 and the profile defined in 3a, players will prefer (σ_1,σ_2) since there is a likelihood of higher payoff arising from the first stage move.

the end of the second stage: namely, $(x,0,1;0,2x,2),(x,0,1;x,2x,2)$ and $(x,0,1;2x,2x,1)$. Now among the three possible moves after $(x,0,1;0,2x,2)$, P_1 stands to gain most if it bids 0. Irrespective of what P_1 bids, P_2 has to bid 0 according to the strategy s_2. Now if P_1 bids 0, its payoff $= z + \delta^2(x/2)$, where z is the payoff at the second stage. Similarly if P_1 bids x, its payoff is $z + \delta^2 c(x)$, while if P_1 bids $2x$ its payoff is $z + \delta^2(-x + c(0))$. Clearly, the best bid for P_1 after $(x,0,1;0,2x,2)$ is 0. After $(x,0,1;2x,2x,1)$, it is x. Thus our problem reduces to comparing the payoff $p_1(s_1, s_2)$ against

(i) $p_1(x,0,1;0,2x,2;0,0)$

(ii) $p_1(x,0,1;x,2x,2;0,0)$, and,

(iii) $p_1(x,0,1;2x,2x,1;x,x)$

We now have

(i) $$\begin{aligned} p_1(x,0,1;0,2x,2;0,0) &= c(x) - \delta c(2x) + \delta^2 \frac{x}{2} \\ &\le c(x) - \delta c(x) + \delta^2 \frac{x}{2} \\ &= c(x)(1-\delta) + \delta^2 \frac{x}{2} \\ &\le \frac{x}{2}(1-\delta) + \delta^2 \frac{x}{2} \\ &= p_1(s_1, s_2). \end{aligned}$$

(ii) $$\begin{aligned} p_1(x,0,1;x,2x,2;0,0) &= c(x) - \delta c(x) + \delta^2 \frac{x}{2} \\ &\le p_1(s_1, s_2) \text{ similarly.} \end{aligned}$$

(iii) $$p_1(x,0,1;2x,2x,1;x,x,1) = c(x) + \delta(-x + c(0)) + \delta^2 c(0).$$

Now $c(0)$ and $c(x)$ are $\le \frac{x}{2}$ so that $-x + c(0) \le -c(x)$.

Therefore the payoff $$\begin{aligned} &\le c(x) - \delta c(x) + \delta^2 c(0) \\ &\le c(x)(1 - \delta + \delta^2) \\ &\le p_1(s_1, s_2). \end{aligned}$$

We have therefore proved that P_1 cannot increase its payoff (against P_2 playing s_2) if it changes its strategy at the first stage and bids x. Exactly the same arguments as above show that it cannot improve its payoff by bidding $2x$ at the first stage. Because of symmetry, P_2

cannot better its payoff by bidding other than 0 in the first stage, if P_1 plays s_1.

Thus our problem passes on to considering strategies t_1 for P_1 where $t_1^1 = 0$ and examining if $p_1(t_1, s_2) > p_1(s_1, s_2)$. The first stage outcomes of (t_1, s_2) are $(0,0,1)$ and $(0,0,2)$. t_1^2 must be defined for both these values. Since there are three choices for each and the choices are independent of each other, there are nine possible pairs.

Let us consider a pair such as $t_1^2(0,0,1) = 0$ and $t_1^2(0,0,2) = x$. The second stage outcomes are therefore $L = (0,0,1;0,2x,2)$ and $R = (0,0,2;x,x,2)$. For each of these outcomes *L*, *R*, there are three possible moves (at the third stage) for P_1. The possible payoffs are the averages $(p_1(L,.) + p_2(R,.))/2$. Since we are interested in testing if these values are $\leq p(s_1, s_2)$, it is enough to consider the maximum values of the payoffs for all the 3rd stage moves. For instance among the three possible 3rd stage outcome i) $(0,0,1;0,2x,2;0,0)$ ii) $(0,0,1;0,2x,2;x,0)$ and, iii) $(0,0,1;0,2x,2;2x,0)$, the outcome i) gives the greatest payoff to P_1. (Generally if there are equal number of wins at the end of the 2nd stage then the best strategy of P_1 at the third stage is to bid 0: if there are two wins for P_1 already then P_1 should bid x: and if there are two losses for P_1 already, then P_1 should bid x).

We therefore reduce our problem to the nine choices – one for each of the nine second stage outcomes. We list these out as (R_1 to R_9 below).

$$R_1 : (0,0,1;2x,2x,1;x,2x,2) \text{ and } (0,0,2;2x,x,1;0,0).$$

Here we have taken the case $t_1^2(0,0,1) = 2x$, $t_1^2(0,0,2) = 2x$. This choice gives us the two second stage outcomes $(0,0,1;2x,2x,1)$ and $(0,0,2;2x,x,1)$. Against the first, the best bid of P_1 is x and against the second it is 0. Therefore the best possible bids for P_1 are as.

$$R_2 : t_1^2(0,0,1) = 2x,\ t_1^2(0,0,2) = x : -$$

$$(0,0,1;2x,2x,1;x,2x,2) \text{ and } (0,0,2;x,x,2;x,x,2)$$

$$R_3 : t_1^2(0,0,1) = 2x,\ t_1^2(0,0,2) = 0 : -$$

$$(0,0,1;2x,2x,1;x,2x,2) \text{ and } (0,0,2;0,x,2;x,x,2)$$

$$R_4 : t_1^2(0,0,1) = 0,\ t_1^2(0,0,2) = 2x : -$$

$$(0,0,1;0,2x,2;0,0) \text{ and } (0,0,2;2x,x,1;0,0)$$

$$R_5 : t_1^2(0,0,1) = 0,\ t_1^2(0,0,2) = x : -$$

$$(0,0,1;0,2x,2;0,0) \text{ and } (0,0,2;x,x,2;x,x)$$

$$R_6 : t_1^2(0,0,1)=0,\ t_1^2(0,0,2)=0 : \text{-}$$

$$(0,0,1;0,2x,2;0,0) \text{ and } (0,0,2;0,x,2;x,x)$$

$$R_7 : t_1^2(0,0,1)=x,\ t_1^2(0,0,2)=x : \text{-}$$

$$(0,0,1;x,2x,2;0,0) \text{ and } (0,0,2;x,x,2;x,x,2)$$

$$R_8 : t_1^2(0,0,1)=x,\ t_1^2(0,0,2)=0 : \text{-}$$

$$(0,0,1;x,2x,2;0,0) \text{ and } (0,0,2;0,x,2;x,x)$$

R_9 : (R is the original outcome (s_1,s_2)).

Note that $t_1^2(0,0,1)=x$ and, $t_1^2(0,0,2)=2x$ is merely the strategy s_1. Again, the 2nd stage outcomes are $(0,0,1;x,2x,2)$ and $(0,0,2;2x,x,1)$. The best third stage moves for P_1 are 0 in each case, and this yields s_1. We calculate the payoff values of the outcomes in each of the above cases and compare them with $p_1(s_1,s_2)$.

R_1:

$$p_1(t_1,s_2)=\left(\frac{p_1(0,0,1;2x,2x,1;x,2x,2)+p_1(0,0,2;2x,x,1;0,0)}{2}\right)$$

$$p_1(s_1,s_2)=\left(\frac{p_1(0,0,1;x,2x,2;0,0)+p_1(0,0,2;2x,x,1;0,0)}{2}\right)$$

In order to compare $p_1(t_1,s_2)$ and $p_1(s_1,s_2)$ it is therefore enough to compare

$$\alpha = p_1(0,0,1;2x,2x,1;x,2x,2) \text{ with, } \beta = p_1(0,0,1;x,2x,2;0,0).$$

$$\alpha = x+c(0)+\delta(-x+c(0))+\delta^2(-c(x)) \text{ and } \beta = x+c(0)-\delta c(x)+\delta^2(x/2)$$

$$\beta-\alpha = -\delta c(x)+\delta^2 c(x)-\delta c(0)+\left(\delta x+\delta^2(x/2)\right)\geq 0 .$$

Similarly it can be proved in a straightforward way that in the cases R_2, R_3, R_4, $p_1(t_1,s_2)\leq p_1(s_1,s_2)$. We need only to draw upon the hypotheses $c(0)\leq c(x)\leq c(2x)$, $c(x)\leq x/2$ and $c(2x)\leq x$.

We turn to the cases R_5, R_6, R_7 and R_8. By assumption, $c(0)\geq (x/2)(2-\delta/2+\delta)$. Now consider R_5.

$$p_1(t_1,s_2)=\frac{p_1(0,0,1;0,2x,2;0,0)+p_1(0,0,2;x,x,2;x,x)}{2}$$

$$=\frac{1}{2}\left(x-\delta c(2x)+\frac{\delta^2 x}{2}-\delta c(0)-\delta^2 c(0)\right)$$

$$p_1(s_1,s_2)-p_1(t_1,s_2)=\left(-\delta x+\frac{\delta^2 x}{2}+\delta c(2x)+\delta c(0)+\delta^2 c(0)\right)\frac{1}{2}$$

$$= (\delta/2)(c(0)+c(2x)+\delta c(0)-x+\delta x/2)$$

≥ 0 by assumption.

This completes the discussion of R_5. R_6 is not much different from R_5.

Let us give the details for R_7. Here $t_1^2(0,0,1)=x$ and $t_1^2(0,0,2)=x$.

The third stage outcomes which have to be considered are $(0,0,1;x,2x,2;0,0)$ and $(0,0,2;x,x,2;x,x,2)$. It turns out that, $p_1(t_1,s_2)=\frac{1}{2}\left(x-\delta c(x)-\delta c(0)-\delta^2 c(0)+\delta^2(x/2)\right)$.

And, it is easily seen that $p_1(t_1,s_2)\geq p_1(s_1,s_2)$ since $c(0)+c(x)+\delta c(0)\geq x(1-\delta/2)$ by hypothesis. The other cases are similarly proved. We have thus proved this theorem.

V. Impact of Differences in the Reputation Functions

Suppose that the reputation functions of P_1 and P_2 are $c_1(\bullet)$ and $c_2(\bullet)$. Even when $c_1(\bullet)$ and $c_2(\bullet)$ are different, the conclusions of theorems 2, 3 and 3a do not vary. The statements of the conditions under which the profiles described are N.E. should now be stated for both c_1 and c_2 separately. The proofs go through without any modification, except when the p_i's are calculated, $c(\bullet)$ should be replaced by $c_i(\bullet)$ $i=1,2$. Also, Theorem 1 stands as it is. We reexamine the implications of theorems 2,3 and 3a.

Let us list out the possible N.E. strategies arising in the theorems 2, 3 and 3a.

(a) $\tau=(\tau_1,\tau_2)$; $\tau_1^r=\tau_2^r=x$, $r=1,2,3$.

In other words, both players bid x, whatever the outcome. This is a N.E. strategy without any preconditions on c_1 or c_2.

(b) $s=(s_1,s_2)$; $s_1^1=s_2^1=0$ and $s_1^r=s_2^r=x$, $r=2,3$ is a N.E. profile, provided

$c_i(x)+\delta c_i(0)+\delta^2 c_i(0)\leq x/2$, $i=1,2$

(c) $\sigma=(\sigma_1,\sigma_2)$ and $\overline{\sigma}=(\overline{\sigma}_1,\overline{\sigma}_2)$ defined respectively in theorems 3 and 3a. These are N.E. profiles provided

$c_i(x)+\delta c_i(0)+c_i(0)\leq \delta x/2$, $\quad i=1,2$

Recall that σ gives $(0,0)$ at the first stage while $\bar{\sigma}$ yields (x, x) at the first stage. In both σ and $\bar{\sigma}$, the winner at the 1st stage elects to bid 0 at the 2nd stage and 0 at the 3rd; while the loser at the 1st stage bids x at the second stage and 0 at the third.

We will find it useful to compare the relative strengths of the conditions under which the strategies can be played. τ can be played in all conditions. The condition for s is weaker than the condition for σ or $\bar{\sigma}$.

Having said the above, suppose c_1 satisfies the condition in (c) while c_2 does not. This means that c_1 satisfies the condition in (b). So may c_2. Thus, P_1 has the option of playing τ_1 or s_1 or σ_1 or $\bar{\sigma}_1$. P_2 cannot play either σ_1 or $\bar{\sigma}_1$, but can play s_2. The question is whether the pair (σ_1, s_2) is a N.E. strategy for the game. Let us look at the final outcome, which is the pair,

$$(0,0,1;0,x,2;0,x,2) \text{ and, } (0,0,2;x,x,2;x,x,2).$$

It is clear that $p_1(s_1, s_2) > p_1(\sigma_1, s_2)$. Look at another example; P_1 can play τ_1 or s_1 or σ_1 or $\bar{\sigma}_1$ while P_2 is a weaker state, and is confined to playing τ_2 only. In this case the only N.E. strategy open to the players is (τ_1, τ_2). Though P_1 is apparently politically stable, it can afford to wait for firms to arrive and, even allow its opponent to win the in 2nd stage. If it itself had a first stage win, P_2 appears to value its reputation effect high with say $c_2(x) > x/2$ so that it opts to bid x in the first stage itself. This forces P_1 also to bid x and x forever.
This is shown in the extensive form in figure 5.

Figure 5 here

All Nash equilibria that result as an outcome of competition between states with unequal political or economic infrastructure will be inefficient. These conclusions have important implications for federal policy. An across the board liberalization policy allowing states with different political and welfare perceptions to bid freely for attracting business, will hurt the better placed states. “One bad egg will make the whole basket go bad”.

VI. Conclusions

The extant analysis of bidding for FDI in federal countries has ignored diversity in the economic conditions of states or at least considered it relatively less important. This paper relaxes this assumption by differentiating among states according to their reputation. It also emphasizes the intertemporal impacts that such diversity can have through reputational effects.

In this paper we develop a substantial generalization of the extant literature on bidding for attracting investment by states in a federal framework by constructing a model of a game (among diverse players) which focuses on the role of "reputation factors" in the design of these bidding processes. This permits us to advance several robust conclusions about the bidding process.

First, we show that the magnitude of the discount factor and the reputation effect have impacts on the possible Nash Equilibrium. Second, we clarify the role that multi-stage bidding has in the outcome of competition for investment funds in a federal framework. We have shown that states can actually bid at some stages and achieve a more efficient equilibrium outcome. Hence, the intertemporal aspect of the game is critical, an issue largely ignored in the extant literature.

Third, we have shown that a state facing a prospect of continual failure in the bidding process may resort to what we term as "suicide strategies" in order to win at a later stage. Suicide strategies bear resemblance to some aspects of economic liberalization. We have shown that in the absence of any tacit understanding between states, a losing state in the initial stage can in fact become a Stackelberg leader in the sense of exercising influence on winning states. We have further shown that the loser by playing suicide strategies can inflict heavy losses on the initial winner.

Finally, by examining the impact of differences in reputation functions across states we are able to make clear the role played by states with weak reputations in affecting the bid sizes of states with high reputation. We show that weaker states can increase the cost of bidding for the stronger states.

This paper has thus emphasized the role that time and diversity across states can play in designing FDI policy. Indeed, the admission of these factors could lead to considerably worse outcomes than those admitted in the extant literature and, at the very least, underscores the importance of national coordination of FDI policy in federal states.

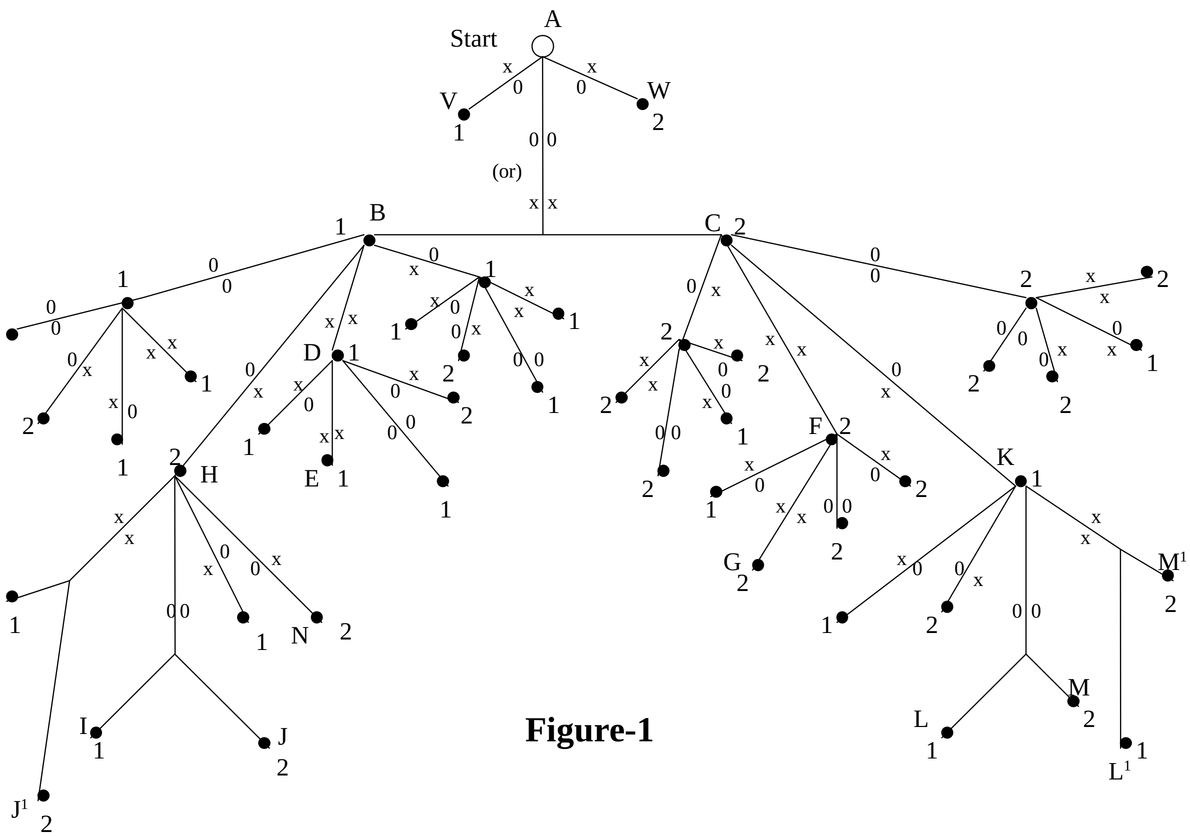
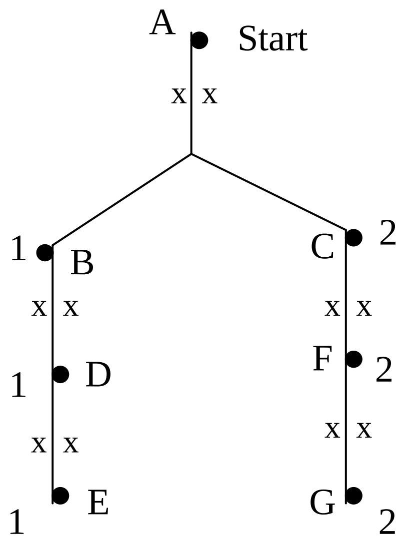


Figure-1

The top node marked A is the starting point. When the bids are equal (0,0) or (x,x), there are two results B and C. B is marked with (1) indicating that P_1 wins & C is marked with (2) to indicate P_2 wins.

The paths are marked with x, 0 etc. on either side. The left (on the page) side indicates the bid of P_1 & the right side that of P_2 (for example in the path AV, P_1 bids x & P_2 bids 0 and therefore P_1 wins at V).

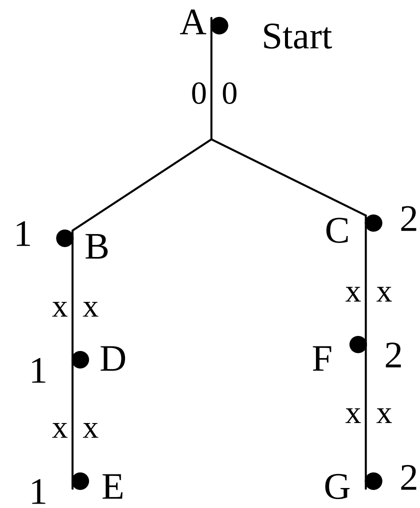
The profile τ



$p_1 = p_2 = 0$

Figure-2a

The profile $s = (s_1, s_2)$

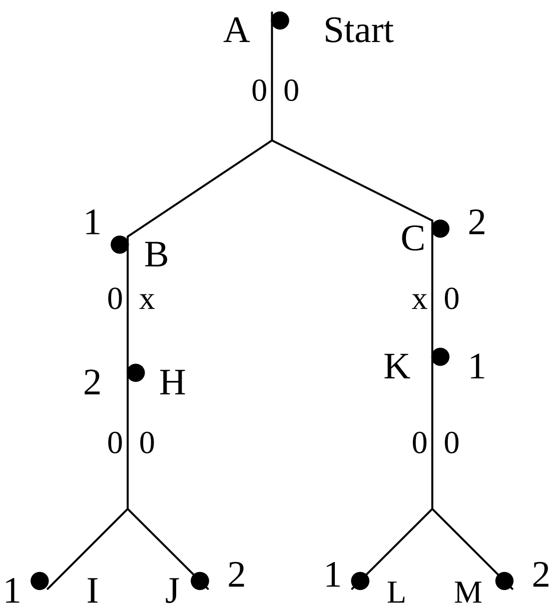


Condition $c(x) + \delta c(0) + \delta^2 c(0) \le x/2$

$p_1 = p_2 = x/2$

Figure-2b

The profile σ

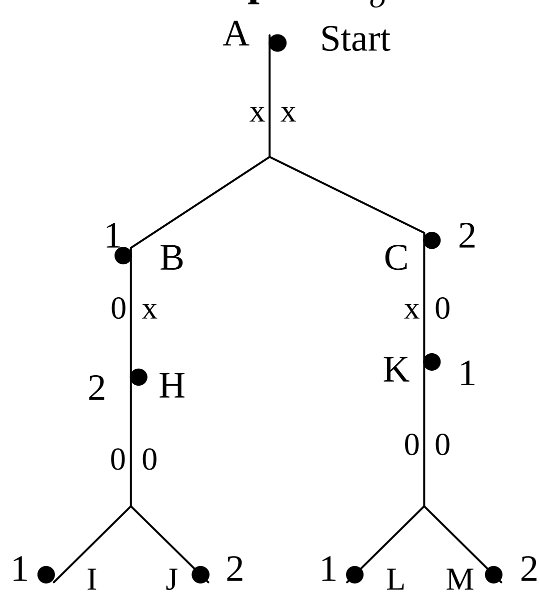


Condition $c(0) + c(x) + \delta\ c(0) \le \delta x/2$

$p_1 = p_2 = x/2\left(1+\delta^2\right)$

Figure-2c

The profile $\overline{\sigma}$

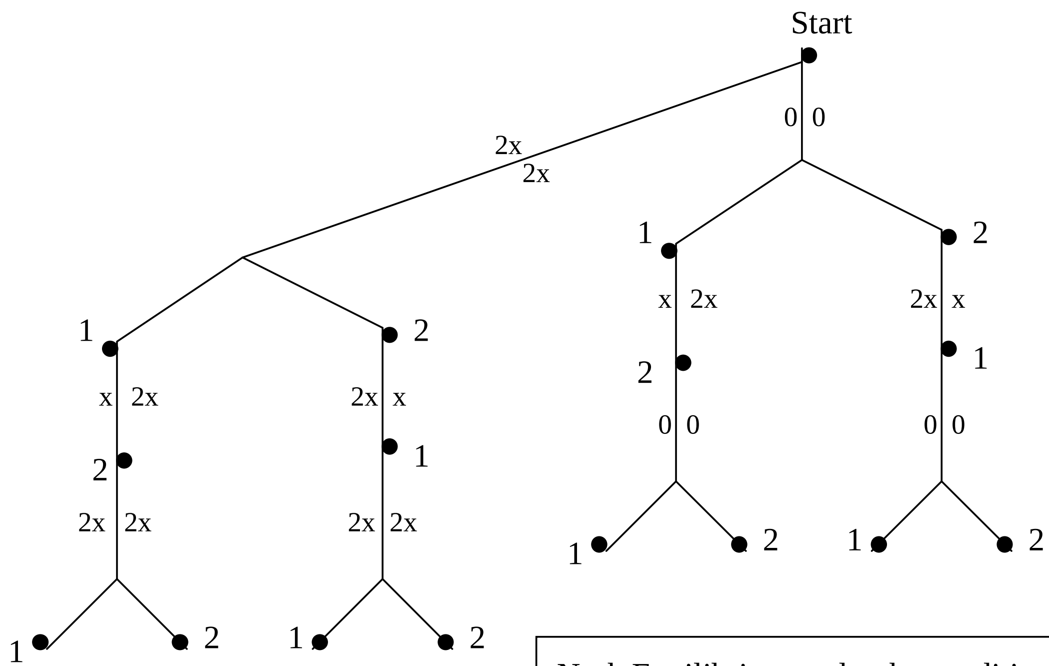


$c(0) + c(x) + \delta\ c(0) \le \delta x/2$

$p_1 = p_2 = \delta^2 x/2$

Figure-2d

Figure 2 : The Possible Nash Equilibrium Problems



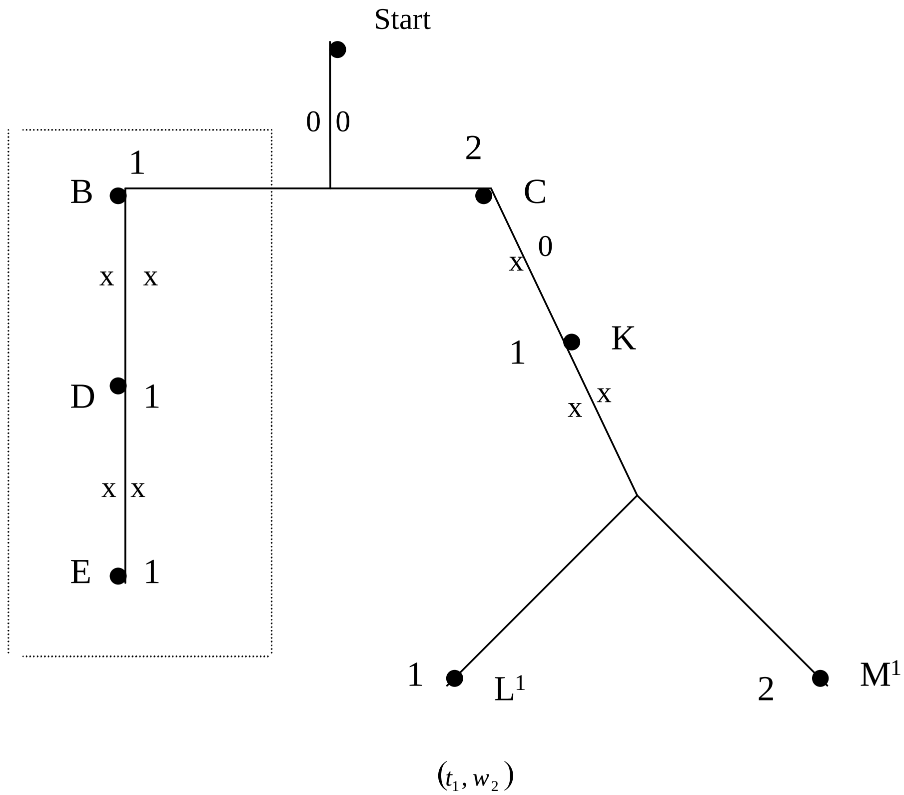
payoff : $p_1 = p_2 = -x(1+\delta^2)$

Nash Equilibrium under the conditions

$c(0)+c(x)+\delta\ c(0) \geq x(1-\delta/2)$

pay off : $p_1 = p_2 = x/2\left(1-\delta+\delta^2\right)$

$c(x) \leq x/2$ and $c(2x) \leq x$



Payoff of P_1 is $1/2\left[x+\delta c(0)+\delta c(x)+\delta^2 c(0)\right] > x/2$

Figure 4

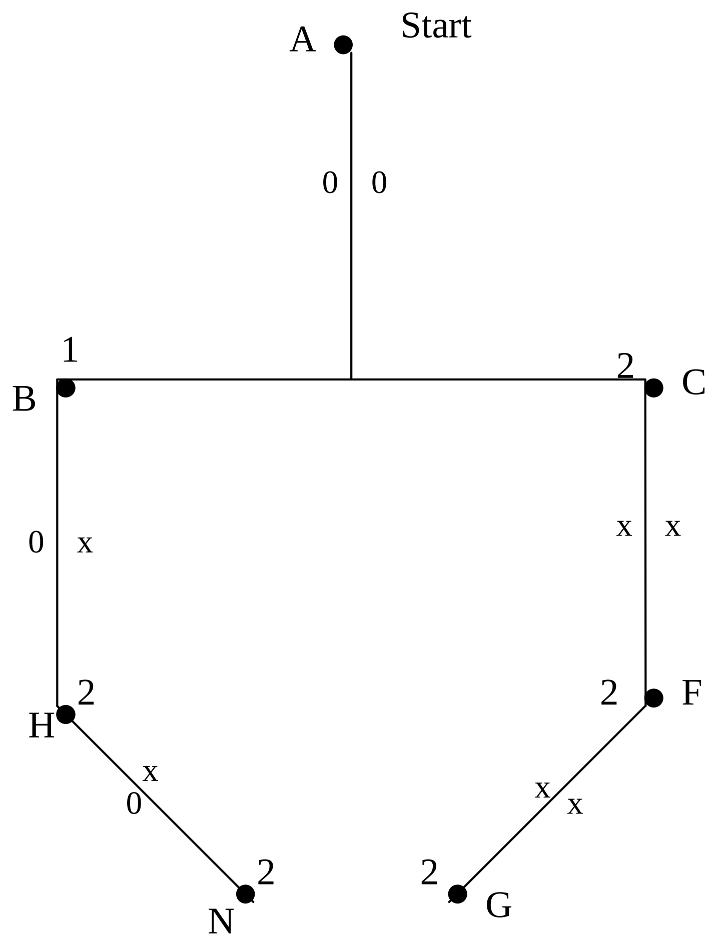


Figure 5: Competition between States with unequal $c(\bullet)$

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