

# **Adjustment costs and the neutrality of income taxes**

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## **Abstract**

A true income tax would not affect asset values or investment decisions for given values of cash flows and pre-tax interest rates (Samuelson, 1964). However, most so-called income taxes do not fully tax capital gains on accrual. This note shows that in the absence of adjustment costs, investment decisions are not distorted by the lack of a comprehensive tax on the capital gains on unimproved land, provided that the depreciation of improvements is allowed as a tax deduction. It also provides the intuition underlying the closely related results of Hartman (1978) and Abel (1983).

**Key words:** Capital gains tax, depreciation, income tax, investment neutrality.

**JEL Code:** H25

## 1. Introduction

Samuelson (1964) showed that a tax on true income, defined as net cash flow plus appreciation of asset values, has no effect on the value of an asset or a firm, holding constant the cash flows in all periods and the pre-tax interest rate. In this sense, a tax on true income is ‘investment neutral’. Of course, it does not follow from Samuelson’s theorem that optimal investment decisions are independent of the rate of a tax on true income. To the contrary, under fairly general assumptions, an income tax reduces savings and therefore, at least in a closed economy, it must also reduce investment. However, to the extent that aggregate savings and investment are both reduced by the imposition of an income tax, this happens because the imposition of an income tax increases the pre-tax interest rate, thereby reducing investment, but lowers the after-tax interest rate, thereby reducing savings.

The present paper deals with income taxes, in which by definition interest receipts are taxed and interest payments are tax deductible.<sup>1</sup> Hartman (1978: 254) appeared to contradict Samuelson’s theorem by claiming that ‘a tax system involving the tax deductibility of both interest payments and of true economic depreciation is neutral if there are no adjustment costs *but is not neutral if adjustment costs are present* (emphasis added)’. We refer to these two findings as Hartman’s ‘positive result’ (that is, neutrality in the absence of adjustment costs) and his ‘negative result’ (that is, the alleged non-neutrality in the presence of adjustment costs).

Abel (1983: 705) claimed that ‘In the presence of costs of adjustment, true economic depreciation is calculated using the shadow price of installed capital ...’, where the shadow value of installed capital is the marginal value to the firm of a unit of installed

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<sup>1</sup> Hartman (1978) and Abel (1983) also discuss tax bases that exclude interest payments and receipts. The basic neutrality result for such ‘cash flow’ taxes is due to Brown (1948). If the tax authorities take a fixed share of all revenues and bear the same fixed share of all costs, they effectively appropriate a corresponding share of the equity in the taxed project or asset. This correspondingly reduces the value of the asset, but does not change the investment decisions that maximize its value.

capital. He showed that in the presence of adjustment costs, investment neutrality can be preserved if the tax authorities allow firms to deduct depreciation defined in this way.

We confirm that Samuelson's theorem does in fact hold quite generally, both in the presence and absence of adjustment costs. This is not an extension of Samuelson's theorem, since his proof used a framework that does not exclude adjustment costs.<sup>2</sup> Rather, by using a framework that explicitly includes adjustment costs of the type modeled by Hartman and Abel we merely clarify the generality of Samuelson's theorem. We then show that it provides the intuition that underlies the results of both Hartman and Abel.

The difference between Samuelson's result, on the one hand, and Hartman's and Abel's, on the other, is due to different assumptions about how the tax authorities measure allowable depreciation—that is, what can be subtracted from cash flow in calculating taxable income. Samuelson's tax base is true economic income, defined as the cash flow minus the fall in the total value of the firm, or asset. Hartman and Abel assume that taxable income is the cash flow minus the fall in the value at market prices (Hartman) or shadow prices (Abel) of the firm's capital.

The value of any firm or project can be thought of as the value of its fixed factors and its variable factors. In the terminology of real estate investment, the former can be labeled 'unimproved land' and the latter 'improvements'. Samuelson effectively defines depreciation as the change in the total value of the unimproved land and the associated improvements, whereas Hartman and Abel effectively assume that depreciation is measured as the change in the value of improvements alone. Samuelson's income tax therefore includes a comprehensive accruals tax on capital gains, or losses, whereas the income tax analyzed by Hartman and Abel could be described as a tax on the income from capital alone.

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<sup>2</sup> Abel (1983: 705) incorrectly asserts that the neutrality results of both Brown (1948) and Samuelson (1964) were derived in models without costs of adjustment. While neither paper explicitly mentions adjustment costs, both are sufficiently general to incorporate them and both neutrality results apply to models with adjustment costs.

In this note we survey and unify the contributions of the three authors and make the following contributions:

1. We clarify the fact that Samuelson's neutrality theorem applies in the presence of adjustment costs, despite the claim to the contrary by Hartman and without the need to use shadow prices rather than market prices, as claimed by Abel.
2. We provide the underlying rationale for Hartman's results. In the presence of adjustment costs, the value of a package of land and improvements cannot be unambiguously separated into the value of the land and the value of the improvements. This is because the value of the land depends on whether improvements have been installed, and the value of what has been installed is generally different from its value before installation. In contrast, in the absence of adjustment costs, the value of the land is independent of the improvements and the value of the improvements is independent of whether or not they have been installed. Since unimproved land is in perfectly inelastic supply, it is always efficiently utilized regardless of how it is taxed, and Samuelson's theorem implies that investment in improvements will be independent of the tax rate provided that the true depreciation of improvements is allowed as a tax deduction.

Hartman's positive result has an important policy implication: if adjustment costs are relatively small, then the lack of a comprehensive tax on the capital gains on appreciating but inelastically supplied factors, such as unimproved inner city land, does *not* lead to overinvestment in associated assets, such as inner city housing, provided that the depreciation of improvements is allowed as a deduction against tax—and the tax systems of the United States, United Kingdom, Australia, New Zealand and doubtless many other countries make a rough attempt to do this.

3. We provide the underlying rationale for Abel's finding: his rule for measuring allowable depreciation for tax purposes is a linear approximation to Samuelson's true economic depreciation. By valuing capital for depreciation purposes at its marginal value to the firm, Abel's rule ensures that the marginal change in his proposed measure of allowable depreciation due to any given change in the investment plans of the firm is equal to the marginal change in Samuelson's

measure of allowable depreciation due to the same change in the investment plans of the firm. Given the same marginal after-tax costs and benefits of investment under the two rules, it follows that the optimal investment decisions of the firm must be the same under the two rules. Since Samuelson's rule is investment neutral (point 1 above) it follows that Abel's must also be investment neutral.

## 2. The basic model

In this section we set out a model of the investment choices facing a firm, which we use in the remainder of the note to substantiate the claims made in the introduction.

We use a discrete time model with the following timing conventions: all cash flows—that is, tax payments, interest payments, and all sales and purchases, including new investment—occur at the end of the period indicated by their subscript; stocks are also measured at the end of the period indicated by their subscript, but immediately after the flows have occurred. It may help to think of period  $t$  as a year in which flows occur during the day of December 31<sup>st</sup> and stocks are measured at 24:00 on December 31<sup>st</sup>. Discounting is important over a full year, but not over the course of one day. Interest receipts are taxable and interest payments tax-deductible. The cash flow at the end of period  $t$ , net of 'allowable depreciation', is taxed at rate  $m_t$ . The way in which the tax authorities measure allowable depreciation is initially left unspecified and so is the physical process by which the capital stock in one period depends on its level in the preceding period and on new gross investment. The value at the end of period  $t$  of a representative firm is given by Fisher's formula:<sup>3</sup>

$$V_t = \sum_{s=1}^{\infty} \frac{[1 - m_{t+s}][F(K_{t+s-1}, I_{t+s}, p_{t+s}, w_{t+s}) - v_{t+s} I_{t+s}] + m_{t+s} D_{t+s}}{\{1 + r_{t+1}[1 - m_{t+1}]\} \{1 + r_{t+2}[1 - m_{t+2}]\} \dots \{1 + r_{t+s}[1 - m_{t+s}]\}}, \quad (1)$$

where:

$V_t$  is the value of the firm at the end of period  $t$ , but immediately after the occurrence of period  $t$  cash flows. It can be thought of as the value of the shares, ex dividend and after retained earnings have been reinvested;

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<sup>3</sup> Round parentheses ( ) are used to indicate functional forms and square [ ], or curly { } parentheses are used to group terms for multiplication, addition, etc.

$r_{t+s}$  is the interest rate during period  $t+s$ ; that is, between the end of  $t+s-1$  and the end of period  $t+s$ ; the discount factor between the end of period  $t+s$  and the end of period  $t+s+1$  is therefore  $1/(1+r_{t+s})$ .<sup>4</sup>

$p_{t+s}$  is the price of output at the end of period  $t+s$ ;

$v_{t+s}$  is the price of new capital goods at the end of period  $t+s$ ;

$w_{t+s}$  is the wage rate at the end of period  $t+s$ ;

$K_{t+s}$  is the capital stock at the end of period  $t+s$ , immediately after the occurrence of gross investment in year  $t+s$ ;

$I_{t+s}$  is gross investment at the end of period  $t+s$ , defined as the number of units of capital purchased and installed;

$F(K_{t+s-1}, I_{t+s}, p_{t+s}, w_{t+s})$  is the maximum cash flow at the end of period  $t+s$ , net of wages, but before subtracting tax and investment spending. The wage rate,  $w_{t+s}$ , is included in the net cash flow function because the firm is assumed to choose employment to maximize the net cash flow, given the capital stock, the rate of investment, the wage rate and the price of output. Gross investment,  $I_{t+s}$ , is included as an argument of the cash flow function to allow for the possibility that, in the presence of adjustment costs, some factors that could otherwise have been used to produce saleable output are instead needed to install new capital equipment;

$m_t$  is the marginal rate of income tax. Tax payments are made at the end of each period;

$D_{t+s}$  is the amount of depreciation allowed by the tax authorities at the end of period  $t+s$ .

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<sup>4</sup> The interest rate and all prices are measured relative to a numeraire that might be money, or some specified good, or bundle of goods. If the nominal interest rate is  $r$  and if the rate of inflation in terms of some specified bundle of goods is  $\pi$ , then a tax on nominal interest at rate  $m$  is equivalent to a tax on real income, in terms of the specified bundle of goods, at rate  $\tau$ , where  $\tau = rm/[r-\pi]$ . Therefore, with interest rates and inflation given and exogenous, the difference between taxing real and nominal income is just a difference in the numerical value of the tax rate.

Since tax in equation 1 is levied on the cash flow net of current investment expenditure it might appear that the tax rules are always assumed to allow the immediate expensing of investment. In fact, however, the formulation is quite general because equation 1 does not specify how investment spending is treated in the definition of  $D_t$ .

Equation 1 can be updated by one period to give an expression for  $V_{t+1}$ . Substitution of this expression back into equation 1, gives:

$$V_t = \frac{[1 - m_{t+1}][F(K_t, I_{t+1}, p_{t+1}, w_{t+1}) - v_{t+1}I_{t+1}] + m_{t+1}D_{t+1} + V_{t+1}}{1 + r_{t+1}[1 - m_{t+1}]} \quad (2)$$

Multiplying both sides of equation 2 by  $1 + r_{t+1}[1 - m_{t+1}]$  and then subtracting  $V_t$  from both sides gives the equation of yield, according to which the value of the enterprise, or asset, multiplied by the after-tax interest rate must equal the after-tax income, defined as the cash flow, plus the capital gain, minus all tax payments:

$$r_{t+1}[1 - m_{t+1}]V_t = [1 - m_{t+1}][F(K_t, I_{t+1}, p_{t+1}, w_{t+1}) - v_{t+1}I_{t+1}] + m_{t+1}D_{t+1} + V_{t+1} - V_t \quad (3)$$

### 3. Samuelson's income tax neutrality theorem

In this section we derive the tax neutrality theorem of Samuelson (1964), according to which a tax on true income leaves the value of every asset unchanged, for given values of its cash flow in all periods and for a given pre-tax interest rate. As claimed in the introduction, this theorem underlies all the other results referred to in this note.

It is clear from equation 3 that in the absence of tax, the interest rate multiplied by the value of the firm would equal the cash flow plus the increase in the value of the firm. It is also clear from equation 3 that in the presence of a tax that reduces the after-tax interest rate from  $r_{t+1}$  to  $[1 - m_{t+1}]r_{t+1}$  and that reduces the cash flow by the same factor,  $[1 - m_{t+1}]$ , the allowable depreciation that make the value of the firm independent must be minus the capital gain:

$$D_{t+s} = -[V_{t+s} - V_{t+s-1}]. \quad (4)$$

Substituting the implied value of  $D_{t+1}$  into equation 3 gives an equation in which every term is multiplied by  $[1 - m_{t+1}]$ . Dividing through by this factor, subtracting  $V_t$  from both sides of the resulting equation and then dividing both sides by  $[1 + r_{t+1}]$  gives:

$$V_t = \frac{F(K_t, I_{t+1}, p_{t+1}, w_{t+1}) - v_{t+1}I_{t+1} + V_{t+1}}{1 + r_{t+1}}. \quad (5)$$

Equation 5, updated by one period, can be used to replace  $V_{t+1}$  by the present value in period  $t+1$  of the net cash flow in period  $t+2$  and the value of the firm in period  $t+2$ . Successive substitutions of this type imply that:

$$V_t = \sum_{s=1}^{\infty} \frac{F(K_{t+s-1}, I_{t+s}, p_{t+s}, w_{t+s}) - v_{t+s}I_{t+s}}{[1 + r_{t+1}][1 + r_{t+2}] \dots [1 + r_{t+s}]}. \quad (6)$$

Equation 6 has been derived while allowing for the possibility that the rates of income tax in each period may vary and may all be non-zero. It is identical to equation 1 in the case when the tax rate is zero in every period. It therefore shows that, for given values of the pre-tax cash flow and pre-tax interest rate, the value of the firm is independent of the income tax rate, or rates. This is, of course, what is predicted by Samuelson's income tax neutrality theorem and it has been derived here in a model that explicitly includes adjustment costs of the type modeled by Hartman and Abel.

Our finding shows that Hartman's negative result—that a tax on true income is not neutral when adjustment costs are present—is at best misleading because the tax that he analyzes is not a tax on the full income of the firm, but only on part of its income: he assumed that allowable depreciation for tax purposes is defined by an equation, like equation 4' below, that involves only the change in the *value of the capital stock* and not the change in the *total value of the firm*.

#### 4. Abel's income tax neutrality theorem

So far, no assumption has been made about how the firm chooses the path of investment and the capital stock. Samuelson's neutrality result holds for any arbitrarily chosen path of these variables. In the remainder of this note, we assume that the firm acts to maximize its value. For simplicity, we assume that capital decays exponentially:

$$K_{t+s} = [1 - \delta]K_{t+s-1} + I_{t+s}. \quad (7)$$

With the level of the capital stock in the previous period pre-determined, we shall treat the firm's investment decision problem in any period  $t$  as the choice of the level of the capital stock at the end of that period,  $K_t$ , and in all future periods, and regard  $I_{t+s}$  not as a



separate variable, but merely as a compact way of writing  $K_{t+s} - [1-\delta]K_{t+s-1}$ . Using  $\tilde{K}_t$  to denote the optimal value of  $K_t$ , true economic depreciation on the optimum path is:

$$D_{t+s} = -[V_{t+s}(\tilde{K}_{t+s}) - V_{t+s-1}(\tilde{K}_{t+s-1})]. \quad (4)$$

Let  $q(K_{t+s}^*)$  be the ‘shadow value’ of installed capital at the end of period  $t+s$ , defined as the marginal value to the firm of an extra unit of installed capital when the actual level of the capital stock at the end of period  $t+s$  is the level that would maximize the value of the firm in the absence of any income tax or, equivalently, in the presence of a tax on true economic income:

$$q(K_{t+s}^*) = \frac{\partial V_{t+s}}{\partial K_{t+s}}. \quad (8)$$

We now drop our assumption that there is a comprehensive tax on the capital gains of the firm and assume instead, following Hartman and Abel, that allowable depreciation for tax purposes in any period depends only on the initial and final levels of the capital stock:

$$D_{t+s} = -[z_{t+s}K_{t+s} - z_{t+s-1}K_{t+s-1}], \quad (4')$$

where  $z_{t+s}$  and  $z_{t+s-1}$  are parameters set by the tax authorities that are exogenous to the firm.

Now consider the first order conditions for the levels of the capital stock in all future periods that maximize the value of the firm in period  $t$  when allowable depreciation is given by equation 4', rather than by equation 4 or  $\tilde{4}$ . The tax authorities' rule for allowable depreciation affects these first order conditions only through its effect on the partial derivatives of allowable depreciation with respect to the capital stock. Since equation 4' implies that  $K_{t+s}$  affects allowable depreciation in periods  $t+s$  and  $t+s+1$ , it follows that the same first order conditions for the capital stock will be obtained when allowable depreciation is given by equation 4', rather than equation  $\tilde{4}$ , if the partial derivatives of  $D_{t+s}$  and  $D_{t+s+1}$  in equation 4' with respect to  $K_{t+s}$  are identical to the partial derivatives of  $D_{t+s}$  and  $D_{t+s+1}$  in equation  $\tilde{4}$  with respect to  $K_{t+s}$ . That is, investment neutrality will be obtained if the parameters used in the tax authorities' formula for allowable depreciation are equal to the shadow prices defined by equation 8:

$$z_{t+s} = q(K_{t+s}^*), \quad (9)$$

$$D_{t+s} = -[q(K_{t+s}^*)K_{t+s} - q(K_{t+s-1}^*)K_{t+s-1}]. \quad (4'')$$

Equation 4'' is the discrete time version of Abel's answer to the question of how  $D_t$  must be set in order to achieve tax neutrality. Abel derived this condition by explicitly analyzing the investment decisions that maximize the value of the firm. Our derivation of it from Samuelson's theorem, and without the need to derive the firm's first conditions provides the intuition that underlies Abel's theorem—namely, his tax rule is a linear approximation to Samuelson's rule and a linear approximation is all that is needed to yield the same first order conditions under the two rules for measuring allowable depreciation—Samuelson's and Abel's.

The present derivation of Abel's rule also demonstrates the need for a caveat that Abel does not explicitly mention: the shadow value of capital used by the tax authorities in each period in equation 4'' must be set so that it is exogenous to the firm and must be evaluated at the level of the capital stock that would be chosen in absence of taxation. If equation 4'' were replaced by:

$$D_{t+s} = -[q(K_{t+s})K_{t+s} - q(K_{t+s-1})K_{t+s-1}], \quad (4''')$$

the firm would generally be able to manipulate the shadow price of capital in each period and the partial derivatives of  $D_{t+s}$  and  $D_{t+s+1}$  with respect to  $K_t$  would no longer be minus  $q(K_{t+s})$  and  $q(K_{t+s})$ , respectively, but would also contain terms in the partial derivative,  $q'(K_{t+s})$ .

Setting allowable depreciation according to equation 4'' is much easier said than done. Abel (1983: 710–711) shows that if the firm's production function is linearly homogenous in capital, labor and investment—thus ruling out the case where unimproved land is important—and if the firm is a price taker, then the shadow value of capital can be estimated as the ratio of the value of the firm to the level of the capital stock. However, this result is of little interest in designing policy since if the value of the firm is observable, there is no need to estimate the shadow value of capital, nor to worry about whether the production function is linearly homogenous or not. Neutrality can be achieved simply by implementing Samuelson's tax on true economic income. Abel concludes his discussion of the case in which the production function is linearly homogenous as follows: 'In this case, the neutral tax system can be stated as follows: expense all capital expenditures and treat any change in the value of the firm as taxable

income (p.711).’ The irony of this is that Abel’s proposal to ‘expense all capital expenditures and treat any change in the value of the firm as taxable income’ is Samuelson’s proposal and, as confirmed in section 3, it works in every case, regardless of whether the production function is linearly homogenous or not.

Hartman’s positive result is useful in designing efficient income tax treatment of assets and firms whose value cannot be readily observed by the tax authorities, provided that adjustment costs are small. If adjustment costs are substantial and if asset values can only be readily observed when assets are traded, the best feasible approximation to a true income tax is probably the proposal of Green and Sheshinski (1978) and Meade *et al.* (1978) that the authorities should levy tax on capital gains only at realization, but should then charge the tax-payer for the estimated cumulated interest cost to the authorities of the delay in payment from accrual to realization. This interest cost would be estimated by using the observed interest rates over the period for which the asset was held and assuming that the appreciation, or depreciation, in its value occurred at a constant exponential rate between its observed purchase and sale dates.

This proposal would approximate Samuelson’s comprehensive income tax, but the approximation would involve errors in the case of an asset whose value did not change at a constant exponential rate between its purchase and sale.

## **5. The investment decision in the general case**

In the previous section we did not derive the first order conditions for the optimal choice of the capital stock in each period, but merely noted that the same first order conditions will be obtained regardless of whether allowable depreciation is set by equation 4 or by equation 4’. In this section we explicitly derive the first order conditions in the general case in which the value of the firm is given by equation 1 and allowable depreciation is given by equation 4’.

It can be seen by inspection of equation 1 that the value of the firm at the end of period  $t$ , when  $K_t$  is given, is independent of the capital stock at the end of the previous period.<sup>5</sup> Since this holds for any period, we have:

$$\frac{\partial V_{t+s}}{\partial K_{t+s-1}} = 0. \quad (10)$$

Next, note that if  $K_{t+1}$  is chosen to maximize  $V_t$ , then:

$$\frac{\partial V_t}{\partial K_{t+1}} = 0. \quad (11)$$

This merely means that the present value of the benefits of an amendment to the firm's investment plans that raises gross investment in period  $t+1$  by one unit and reduces gross investment in period  $t+2$  by  $[1-\delta]$  units, so as to raise  $K_{t+1}$  by one unit and keep  $K_{t+2}$  unchanged, must equal the present value of the costs of this amendment. Equation 11 implies that the partial derivative of the numerator on the right side of equation 2 with respect to  $K_{t+1}$  must be zero. But  $K_{t+1}$  only enters this numerator through  $V_{t+1}$ ,  $D_{t+1}$  and  $I_{t+1}$ , which is an abbreviation for  $K_{t+1} - [1-\delta]K_t$ . Therefore:

$$\frac{\partial V_{t+1}}{\partial K_{t+1}} \equiv q(K_{t+1}) = [1 - m_{t+1}][v_{t+1} - F_I(t+1)] + m_{t+1}z_{t+1}. \quad (12)$$

where  $F_I(t+1)$  denotes the partial derivative of the cash flow in period  $t+1$  with respect to  $I_{t+1}$ . Next, partially differentiate equation 2 with respect to  $K_t$ , recalling that  $\partial V_{t+1}/\partial K_t$  is zero and that  $D_{t+1}$  is here assumed to be given by equation 4':

$$q(K_t) = \frac{[1 - m_{t+1}]\{F_K(t+1) - [1 - \delta][F_I(t+1) - v_{t+1}]\} + m_{t+1}z_t}{1 + r_{t+1}[1 - m_{t+1}]}, \quad (13)$$

where  $F_K(t+1)$  denotes the partial derivative of the cash flow in period  $t+1$  with respect to  $K_{t+1}$ .

In the next two sections we consider the special cases of equations 12 and 13 that correspond to the assumptions of Hartman and Abel.

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<sup>5</sup> Equation 1 shows that  $V_t$  does depend on both  $D_{t+1}$  and  $I_{t+1}$ . However, we have assumed that allowable depreciation is given by equation 4'. This makes  $D_{t+1}$  independent of  $K_{t-1}$ . Similarly, equation 7 makes  $I_{t+1}$  independent of  $K_{t-1}$  for given values of  $K_t$  and  $K_{t+1}$ .

## 6. Hartman's income tax neutrality theorem

In the absence of adjustment costs,  $F_I(t)$  is zero for all  $t$ . Hartman showed that investment neutrality can be preserved in this special case by setting  $z_{t+s} = v_{t+s}$ , for all  $s$ . This result can be readily derived from equations 12 and 13. In the absence of adjustment costs,  $F_I(t) = F_I(t+1) = 0$ . With  $z_{t+1} = v_{t+1}$  equation 12 therefore becomes:

$$q(K_{t+1}) = v_{t+1} \quad (12')$$

and with  $z_t = v_t$  equation 13 becomes:

$$q(K_t) = \frac{[1 - m_{t+1}]\{F_K(t+1) + [1 - \delta]v_{t+1}\} + m_{t+1}v_t}{1 + r_{t+1}[1 - m_{t+1}]} \quad (13')$$

Using equation 12' lagged one period to replace the left side of equation 13' by  $v_t$ , gives:

$$v_t = \frac{[1 - m_{t+1}]\{F_K(t+1) + [1 - \delta]v_{t+1}\} + m_{t+1}v_t}{1 + r_{t+1}[1 - m_{t+1}]} \quad (14)$$

Multiply both sides of equation 14 by  $1 + r_{t+1}[1 - m_{t+1}]$ , subtract  $m_{t+1}v_t$  from both sides, divide all through by  $[1 - m_{t+1}]$  and rearrange terms to show that the first order condition relating the marginal product of capital to the cost of capital is independent of the rate of income tax:

$$F_K(t+1) = r_{t+1}v_t + \delta v_{t+1} - [v_{t+1} - v_t] \quad (15)$$

The three terms on the right side of equation 15 correspond to the interest cost of capital, physical depreciation and capital losses, before allowing for physical depreciation. The sum of these three terms is the cost of capital as derived by Jorgensen (1965) and others.

The intuition behind Hartman's theorem—that in the absence of adjustment costs a tax on capital income alone that does not include a comprehensive tax on capital gains is investment neutral—can be seen by noting that in the absence of adjustment costs, it makes no difference to the firm whether it rents its capital each period, or buys it and installs it. Suppose that it rents it from a firm that owns capital, but does not own any land. First, note that if the capital-owning firm is allowed to deduct true economic depreciation, it will make a normal rate of return on its capital if it charges the same rental rate as it would have charged in the absence of taxation. Next, note that the value of the land-owning firm, with all capital rented, is simply the value of the land that it owns. This will depend on the stream of future revenues—net of the cost of renting capital and other factors—that could be obtained if the optimal amounts of capital and

other factors were employed in each future period. This value is independent of the amount of capital currently employed. In this situation, there is therefore no scope for altering the amount of capital gains on some particular parcel of land by over-investing in the capital employed with it. The land-owning firm's decision on how much capital to hire each period is exactly analogous to its decision on how much labor to hire. Provided that expenditure on renting capital is allowed as a deduction against tax, the land-owning firm will maximize profits in each period by equating the marginal product of capital to its per period rental price, just as it equates the marginal product of labor to the pre-tax wage rate per period. Since the income tax does not alter the per period rental price of capital corresponding to any particular value of the interest rate, it has no effect on the level of investment that equates the marginal product of capital to the per period rental price of capital.

The above logic generally breaks down if adjustment costs are important, since the value of the land owned by the firm is then not independent of the amount of capital that has been installed on it.

## 7. Investment neutrality in the presence of adjustment costs

Abel's theorem that investment neutrality can be preserved, even when  $F_I(t+s)$  is not zero, by setting  $z_{t+s} = q(K_{t+s}^*)$ , for all  $s$ , was derived in section 4 without explicitly deriving the first order conditions by noting that Samuelson's tax ensures investment neutrality by making the firm's value independent of the tax rate and Abel's proposal yields the same first order conditions as Samuelson's by making allowable depreciation a linear approximation to allowable depreciation under Samuelson's tax. Abel's result can now be confirmed by noting that when allowable depreciation is given by equation 4', equation 12 becomes:

$$q(K_{t+1}) = [1 - m_{t+1}][v_{t+1} - F_I(t+1)] + m_{t+1}q(K_{t+1}^*), \quad (12'')$$

and equation 13 becomes:

$$q(K_t) = \frac{[1 - m_{t+1}]\{F_K(t+1) - [1 - \delta][F_I(t+1) - v_{t+1}]\} + m_{t+1}q(K_t^*)}{1 + r_{t+1}[1 - m_{t+1}]}. \quad (13'')$$

By definition, when  $m_t = 0$  for all  $t$ ,  $K_t = K_t^*$  and  $K_{t+1} = K_{t+1}^*$  for all  $t$ . Equations 12'' and 13'' can now be solved for  $q(K_{t+1}^*)$  and  $q(K_t^*)$ , respectively:

$$q(K_{t+1}^*) = [v_{t+1} - F_I^*(t+1)], \quad (12''')$$

and:

$$q(K_t^*) = \frac{F_K^*(t+1) - [1 - \delta][F_I^*(t+1) - v_{t+1}]}{1 + r_{t+1}}. \quad (13''')$$

By replacing  $q(K_{t+1}^*)$  in equation 12'' and  $q(K_t^*)$  in equation 13'' by the expressions on the right sides of equations 12''' and 13''' and rearranging terms it is possible to obtain:

$$q(K_{t+1}) - q(K_{t+1}^*) = [1 - m_{t+1}][F_I^*(t+1) - F_I(t+1)], \quad (12''')$$

and:

$$q(K_t) - q(K_t^*) = \frac{1 - m_{t+1}}{1 + r_{t+1}[1 - m_{t+1}]} [F_K(t+1) - F_K^*(t+1) - [1 - \delta][F_I(t+1) - F_I^*(t+1)]] \quad (13''')$$

It is clear that these two equations can be satisfied by setting  $K_t = K_t^*$  and  $K_{t+1} = K_{t+1}^*$ , so that each of the terms  $q(K_t)$ ,  $q(K_{t+1})$ ,  $F_K(t+1)$  and  $F_I(t+1)$  is equal to the corresponding asterisked term. This makes both sides of both of the above equations zero.

The above demonstration confirms that investment neutrality can be obtained, even in the presence of adjustment costs, by setting allowable depreciation according to equation 4'. Abel's derivation is less cumbersome than the one just given because he ignores the difference between asterisked and unasterisked values and cancels out the terms in  $1 - m_{t+1}$  in equations 12'' and 13''. This gives the right answer, provided one remembers that the values of  $q_t$  and  $q_{t+1}$  that are used by the tax authorities in setting  $z_t$  and  $z_{t+1}$  must be set independently of  $K_t$  and  $K_{t+1}$  and must somehow be evaluated at the levels that would be optimal in the absence of taxation.

Combining equations 12''' and 13''' gives the following first order condition for the optimal choice of capital in period  $t$  in the absence of taxation, or when depreciation is arranged so that taxation is investment neutral:

$$F_K^*(t+1) = r_{t+1}v_t + \delta v_{t+1} - [v_{t+1} - v_t] + [1 - \delta]F_I^*(t+1) - [1 + r_{t+1}]F_I^*(t). \quad (15')$$

Both equation 15 and equation 15' are first order conditions for the optimal level of the capital stock when the tax system is investment neutral. The difference between them is that equation 15' allows for costs of adjustment whereas equation 15 assumes that costs of adjustment are negligible. Both equations compare the costs and benefits in period  $t+1$  of an amendment to firm's investment plans that raise the capital stock in period  $t$  by unit, leaving it unchanged in other periods; this is done by increasing gross investment by one unit in period  $t$  and reducing it by  $[1-\delta]$  units in period  $t+1$ . When adjustment costs are not negligible, this results in the inclusion of two terms on the right side of equation 15' that are absent from equation 15: adjustment costs are raised by  $[1-\delta]F_l^*(t+1)$  in period  $t+1$  and reduced by  $F_l^*(t)$  units in period  $t$ ; to obtain the present value of this latter cost saving in period  $t+1$ , it is necessary to multiply it by  $[1+r_{t+1}]$ .

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